1

Title: Classical Electrodynamics

Outline :

Maxwell's Equations in vacuum

(1) $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ (3) $\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(2) $\vec{\nabla} \cdot \vec{B} = 0$ (4) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

→ • Remark about ϵ_0, μ_0

• notes : - ρ scalar quantity
- $\vec{E}$, $\vec{B}$, $\vec{J}$ are vector quantities
(3 - D) Vectors

Also ∇ is vector

* Maxwell's Equs. are PDE
(2) and (3) are homogeneous
(1) and (4) nonhogeneous → ρ/ϵ_0
↘ $\mu_0 \vec{J}$

what to study ?
- Solution of Maxwell's Equs, under static conditions $\frac{\partial}{\partial t}$ i.e there are not $\vec{B}$ and $\vec{E}$ to vary.

- BVP's: (1) and (3) electrostatic BVP's eqns., (2) and (4) magnetiostatic BVP's

Title:

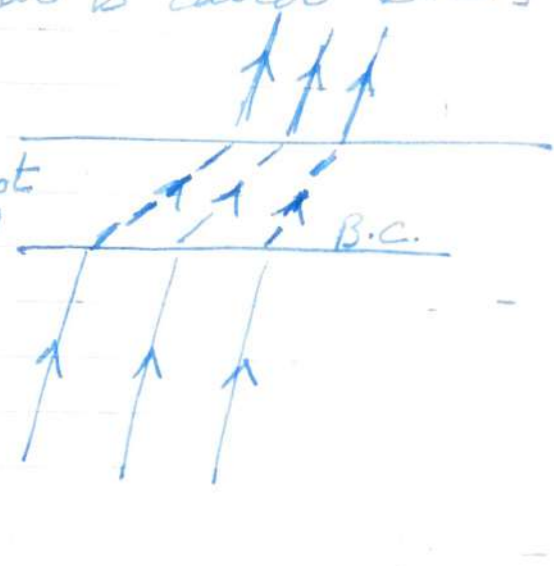
مثال هل يمكن تعيين E باستخدام معادلة ① الإجابة لا
لأنه لا تحمل كامل المعلومات عن المجال الكهربي ولذلك تحتاج
إلى ③ حيث تحتاج div مع curl لمعرفة المعلومات
بالكامل حول أي متغير فيزيائي كالمجال الكهربي $\vec{E}$
كالمجال المغناطيسي $\vec{B}$

PART II Influence of material media

1) Dielectric — Conductors — magnetic material

BVPs in presence of material media give rise to what is called B.C.'s

material not vacuum



Maxwell's Equs., BVP's subject to B.C.

Title:

PART II: Dynamics case: time-varying $\vec{E}$, $\vec{B}$

o E.M. waves propagation in vacuum.

oo E.M. waves ~ ~ material media

(1) conductors

(2) Dielectrics...

PART III: Covariant formation of ED

(i) Gauge invariance

(ii) Relativity and EM

(iii) Lorentz scalars in EM

Ref. (1) Griffiths : Electrodynamics 4th

(2) Jackson: classical ED 3rd ed.

SI

(3) Sadiku: elements of EM

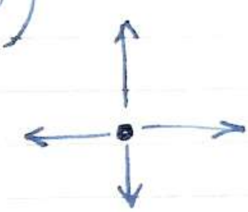
electromatics

Title:

- 1st concept

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{\tau} \frac{\rho(r')}{|\vec{r}-\vec{r}'|^2}(\widehat{\vec{r}-\vec{r}'})\,d\tau'$$

– Due to spherical symmetry of $\vec{E}$ field of a point charge, we use spherical polar coordinate system (r, θ, ϕ)



let $\vec{r}-\vec{r}' = \vec{\mathscr{r}}$

Then $\widehat{\vec{r}-\vec{r}'} = \hat{\mathscr{r}}$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \rho(r')\,\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

Therefore, the fun., $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$ is of interest to use $\therefore f(\mathscr{r}) = \hat{\mathscr{r}}/\mathscr{r}^2$ where $\mathscr{r}$ can represent in spherical coordinates.

Note: Ex. $\mathscr{r}^2 = |\vec{r}-\vec{r}'|^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$

Title:

? let's understand $f(r) = \frac{\hat{r}}{r^2}$

?? Define Gauss-divergence theorem

$$\int_{\text{volume}} \bar{\nabla} \cdot \bar{f} \, dv = \int_{\text{surface}} f \cdot ds$$

note $\vec{\nabla} \equiv i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial k}$ this operates or acting on unprimed variables

$$\frac{\partial}{\partial x}(x - x') = \frac{\partial x}{\partial x} = 1$$

$$\therefore LHS = \int_{\text{volume}} \vec{\nabla} \cdot \vec{f} \, dv = \int_{\text{volume}} \nabla \cdot \frac{\hat{r}}{r^2} \, dv$$

→ dv means small volume i.e. $dx\,dy\,dz$ in cartesian coordinate

→ use r instead of r because $\vec{\nabla}$ acting on $\vec{r}$ not $\vec{r}'$

Title:

→ evaluation of divergence $\frac{\hat{r}}{r^2}$

$\hat{r} \doteq$ unit vector $\equiv \hat{a}_r$

and

$\frac{1}{r^2}$ is a scalar quantity

$$\therefore \vec{u} = \frac{\hat{r}}{r^2} = \underbrace{\frac{1}{r^2}}_{u_r} \hat{a}_r + \underbrace{0}_{u_\theta} \cdot \hat{a}_\theta + \underbrace{0}_{u_\phi} \cdot \hat{a}_\phi$$

Hence

$$\vec{\nabla} \cdot \vec{u} = \frac{1}{r^2}(r^2 u_r)_r + \underbrace{\frac{1}{r\sin\theta}(\sin\theta\, u_\theta)_\theta}_{\text{zero}}$$

$$+ \underbrace{\frac{1}{r\sin\theta}(u_\phi)_\phi}_{\text{zero}}$$

$$\vec{\nabla} \cdot \vec{u} = \frac{1}{r^2}\left(r^2 \frac{1}{r^2}\right)_r = \text{zero}$$

LHS = zero $\quad \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) = 0$ OR $\vec{\nabla} \cdot \left(\frac{\hat{\Omega}}{\Omega^2}\right) = 0$

<u>Now</u>: come to RHS of G.D Theorem

$$RHS = \int_{\text{surface of sphere}} \left(\frac{\hat{r}}{r^2}\right) \cdot \left(r^2 \sin\theta \, d\theta \, d\phi \, dr\right) = 4\pi$$

Title:

LHS = 0 RHS = 4π

this means we're trying to say that G.D Theorem is not valid for the function

$\vec{u} = \frac{\hat{r}}{r^2}$

However G.D Theorem is supposed to be Valid for any vector valued function?

So what to do?

this a paradox.

$\vec{E} \propto \frac{q}{r^2}\hat{r}$ also $f \propto \hat{r}\, q_1 q_2/r^2$

والصبح الـــالم

Definition of Dirac-Delta function

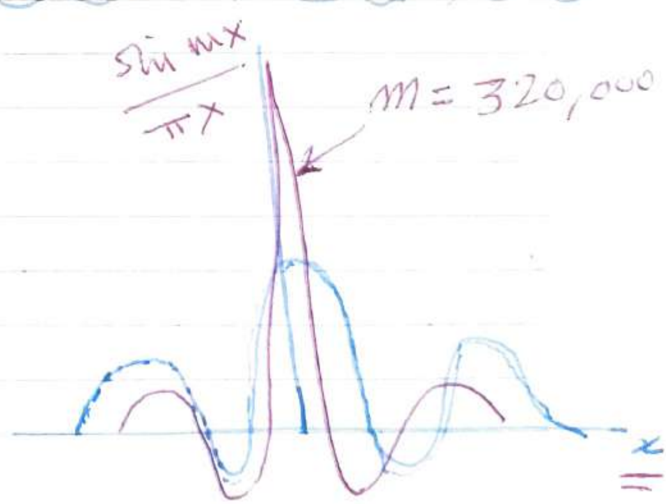
$f(x) = \frac{\sin mx}{\pi x}$

m: parameter

$f(x) = \lim_{m \to \infty} \frac{\sin(mx)}{\pi x}$

this is equivalent to

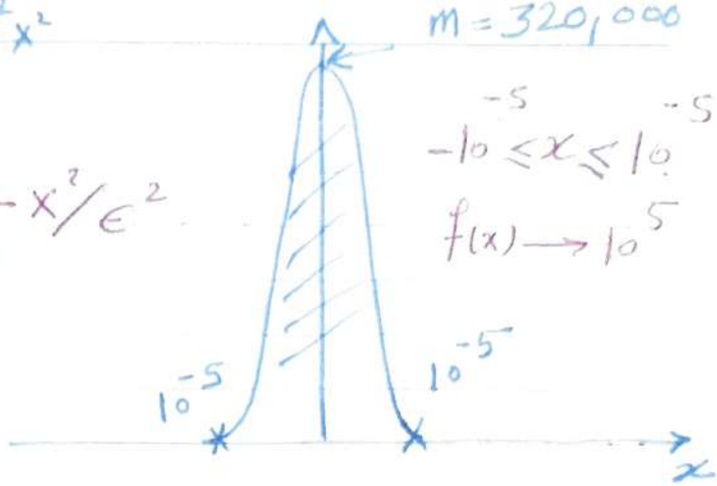
$f(x) = \lim_{\epsilon \to 0} \frac{\sin(\frac{x}{\delta})}{\pi x}$



Title:

$$g(x) = \lim_{m \to \infty} \frac{m}{\sqrt{\pi}} e^{-m^2 x^2}$$

$$g(x) = \lim_{\epsilon \to 0} \frac{1}{\epsilon \sqrt{\pi}} e^{-x^2/\epsilon^2}$$



$m = 320{,}000$

$-10^{-5} \le x \le 10^{-5}$

$f(x) \to 10^{5}$

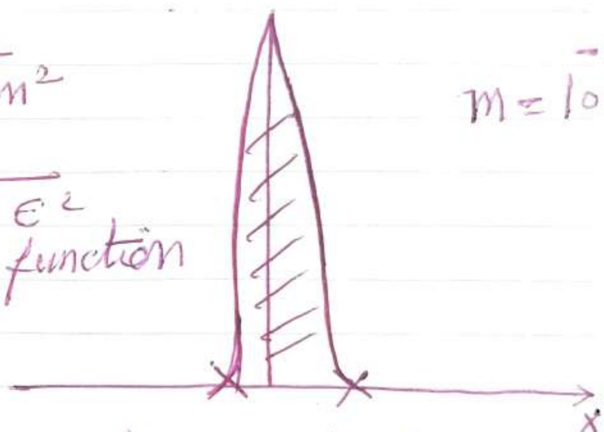
Also

$$f(x) = \frac{m}{\pi} \frac{1}{x^2 + m^2}$$

$$h(x) = \lim_{\epsilon \to 0} \frac{\epsilon}{\pi} \frac{1}{x^2 + \epsilon^2}$$

$m = 10^{-8}$

This is lorentz-like function



As $\epsilon \to 10^{-8}$

$-10^{-9} \le x \le 10^{-9}$ we have a sharply peak function

Why do we need these types of function?

Ex. An electron is located at $x = 0$,
How to write this mathematically?
if you just write q, then the information about its position is missing

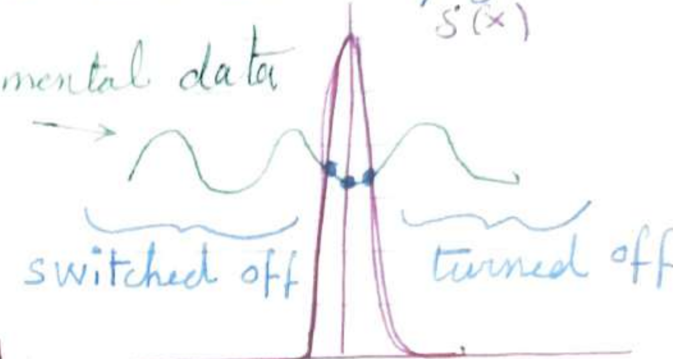
let's denote this sharply peak function $\delta(x - x_0)$

Title:

Exs: $\lim_{\epsilon \to 0} \frac{1}{2\epsilon} e^{-|x|/\epsilon}$, $\lim_{\epsilon \to 0} \frac{\operatorname{Sech}^2(\frac{x}{\epsilon})}{2\epsilon}$

o sharply peaking function come to physics $\underline{\delta(x)}$

? what is the ~~function~~ action of δ-function on some data curve?



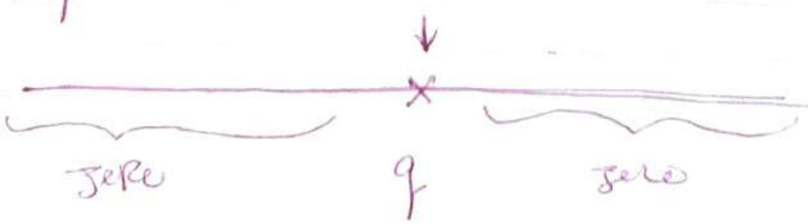
$-10^{-9} \le x \le 10^{-9}$

Ans the Delta-function picks up signal only where δ-peaks (around $x=x_0$) and it rejects all other parts of the signal

$$\Rightarrow \int_{-\infty}^{\infty} \delta(x-x_0)\, f(x)\, dx = f(x_0)$$

Therefore, if we write q at $x=x_0$ then $\equiv q\,\delta(x-x_0)$

where $q\,\delta(x-x_0)$ is zero everywhere except x_0



Title:

∴ Hence, $\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 0$,

$$\int_{vol} \vec{\nabla} \cdot \frac{\hat{r}}{r^2} \, d^3r = 0 \qquad \int_{surf.} \frac{\hat{r}}{r^2} \, \vec{ds} = 4\pi$$

LHS surf. RHS

we can resolve this paradox by saying

$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi \, \delta^3(r) \rightarrow \begin{matrix} 4\pi & \text{at } r=0 \\ 0 & \text{other} \end{matrix}$$

$$= 4\pi \, \delta(x) \, \delta(y) \, \delta(z)$$

Equivalently $\vec{\nabla} \cdot \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 4\pi \, \delta^3(\vec{\mathscr{r}})$

therefore, lets evaluate $\vec{\nabla} \cdot \vec{E}$

$$\vec{\nabla} \cdot \left\{ \frac{1}{4\pi\epsilon_0} \int_{\mathscr{r}'} \rho(\mathscr{r}') \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \, d^3\mathscr{r}' \right\}, \quad \vec{\mathscr{r}} = \vec{r} - \vec{r}'$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \int \rho(\mathscr{r}') \left(\vec{\nabla} \cdot \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \right) d^3r'$$

acting only on $\vec{r}$ unprimed

$$\frac{1}{4\pi\epsilon_0} \int_{\mathscr{r}'} \rho(r') \, 4\pi \, \delta^3(\vec{\mathscr{r}}) \, d^3r'$$

Title:

$$\frac{1}{\epsilon_0}\int_{\Omega'} \rho(r')\,\delta^3(r-r')\,d^3\vec{r}'$$

use $$\int_{-\infty}^{\infty} \delta(x-x_0)\,f(x)\,dx = f(x_0)$$

$$\frac{1}{\epsilon_0}\int \rho(\vec{r}')\,\delta^3(\vec{r}'-\vec{r})\,d^3\vec{r}' = \frac{1}{\epsilon_0}\rho(\vec{r})$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho(\vec{r})}{\epsilon_0}}$$

using

note that δ is symmetrical function
is even

$$\delta^3(\vec{r}'-\vec{r}) = \delta(r-r')$$

Evalution of $\vec{\nabla}\times\vec{E}$

For this, we need to Expand $\left(\vec{\nabla}\wedge\frac{\hat{\mathcal{R}}}{\mathcal{R}^2}\right)$ in Spherical coordenate system

$$\vec{\nabla}\times\frac{\hat{\mathcal{R}}}{\mathcal{R}^2} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{1}{(r-r')^2} & r\cdot 0 & r\sin\theta\cdot 0 \end{vmatrix}$$

$$\vec{\nabla}\times\frac{\hat{\mathcal{R}}}{\mathcal{R}^2} = 0$$

$$\vec{\nabla}\wedge\vec{E} = 0 \equiv \frac{1}{4\pi\epsilon_0}\int_{\Omega'} \rho(\vec{r}')\left(\vec{\nabla}\wedge\frac{\hat{\mathcal{R}}}{\mathcal{R}^2}\right)d^3\vec{r}'$$

where $\frac{\hat{\mathcal{R}}}{\mathcal{R}^2} = \frac{1}{\mathcal{R}^2}\hat{a}_{\mathcal{R}} + 0\,a_\theta + 0\,a_\phi$

check for Updates

Title: How to ~~move~~ make move from statics → dynamics

electrostatics → $\rho(\vec{r})$

magnetostatics → $\vec{J}(r)$ → steady-state motion OR uniform current flow

i.e ① no acceleration OR deacceleration

② no fluctuation

$$\vec{\nabla} \circ \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \circ \vec{B} = 0$$

$$\nabla \wedge \vec{E} = 0 \qquad \vec{\nabla} \times B = \mu_0 \vec{J}$$

valid only for electrostatics (vaccum OR air) valid only for magnetostatics

- note: $\vec{\nabla} \circ \vec{B} = 0$ because no magnetic monople

- Maxwell's equ., is not sufficient, we have or must have additional information

$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$ Electric displacement

$\vec{B} = \mu_0 \mu_r \vec{H}$ magnetic displacement

$\vec{F} = q\left[\vec{E} + \vec{v} \wedge \vec{B}\right]$ Lorentz force

$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ continuity Equation

Title:

linear ?

- what is meant by {homogeneous + istropic}

$\therefore \epsilon_0 \epsilon_r$ and $\mu_0 \mu$ absorpted in $\vec{D}, \vec{H}$

$\vec{\nabla} \cdot \vec{D} = \rho_{free}$ $\qquad$ $\vec{\nabla} \cdot \vec{H} = 0$

$\vec{\nabla} \wedge \vec{D} = 0$ $\qquad$ $\vec{\nabla} \wedge \vec{H} = \vec{J}$

Above Equns. in integral form

$\oint \vec{D} \cdot \vec{ds} = Q_{free}$ $\qquad$ $\oint \vec{H} \cdot \vec{ds} = 0$

$\oint \vec{D} \cdot \vec{dl} = 0$ $\qquad$ $\oint \hat{H} \wedge \vec{dl} = J$

note: With $\vec{\nabla} \cdot \vec{E} \Rightarrow$ surface integral

$\vec{\nabla} \times \vec{E} \Rightarrow$ line integral

Electrostatics	magnetostatics
$\rho(\vec{r}), \vec{E}(\vec{r}), V(\vec{r})$.	$\vec{J}(\vec{r}), \vec{B}(\vec{r}), \vec{A}(\vec{r})$
Given any one quantity find the other two	Given any one of the above, determine the other two
$\rho(\vec{r})$ is the Source (cause)	$\vec{J}(\vec{r})$ cause
$\vec{E}(\vec{r})$ or $V(\vec{r})$ is the effect	$\vec{B}(r), \vec{A}(r)$ are effects
	$\vec{A}(\vec{r})$ is magnetic vector potential

Title:

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
 & \rho \arrow[dl, bend right, "V=\frac{1}{4\pi\epsilon_0}\int\frac{\rho}{\mathscr{r}}\,d\tau"'] \arrow[dr, bend left, "\vec{E}=\frac{1}{4\pi\epsilon_0}\int\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\rho(\vec{r}\,')\,d\tau"] & \\
V \arrow[ur, "\nabla^2 V=-\rho/\epsilon_0"] \arrow[rr, bend left, "\vec{E}=-\vec{\nabla}V"] & & \vec{E} \arrow[ul, "\vec{\nabla}\cdot\vec{E}=\rho/\epsilon_0,\ \vec{\nabla}\times\vec{E}=0"'] \arrow[ll, bend left, "V=-\int\vec{E}\cdot d\vec{l}"]
\end{tikzcd}
$$

For Magnetostatics

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
 & \vec{J} \arrow[dl, bend right, "\vec{A}=\frac{\mu_0}{4\pi}\int\frac{\vec{J}}{\mathscr{r}}\,d\tau"'] \arrow[dr, bend left, "\vec{B}=\frac{\mu_0}{4\pi}\int\vec{J}\,\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau"] & \\
\vec{A} \arrow[ur, "\nabla^2\vec{A}=-\mu_0\vec{J}"] \arrow[rr, "\vec{B}=\vec{\nabla}\times\vec{A},\ \vec{\nabla}\cdot\vec{A}=0"'] & & \vec{B} \arrow[ul, "\vec{\nabla}\times\vec{B}=\mu_0\vec{J},\ \vec{\nabla}\cdot\vec{B}=0"']
\end{tikzcd}
$$

with $\mathscr{r} = r - r'$

unique theorem $\rightarrow$?? $\vec{\nabla}\cdot\vec{A}=0$, $\nabla^2\vec{A}=-\mu_0\vec{J}$ Related to Gauge transformation

Title:

Helmholtz theorem

let $\vec{C}$ be such that

$\vec{\nabla} \cdot \vec{C} = 0$

let g be a scalar function $g(x, y, z)$. then there exists a vector function $\vec{F}$ such that

$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$

$\nabla \wedge \vec{E} = 0$

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$\vec{\nabla} \cdot \vec{F} = g$

$\vec{\nabla} \times \vec{F} = \vec{C}$ $\Rightarrow$ If $|\vec{F}| \to 0$ as $r \to \infty$ then $\vec{F}$ is a unique. Also $|\vec{C}| \to 0$ and $g \to 0$ as $r \to \infty$.

$\vec{C}$ and g are given

Title:

?? why the continuity Equ., is not an independent equation ?? $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

start with $\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}$ Ampere's Law

take divergence

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot (\mu_0 \vec{J})$$

$$\Downarrow$$

$$0 = \mu_0 (\vec{\nabla} \cdot \vec{J})$$

$\vec{\nabla} \cdot \vec{J} = 0$ Continuity Equ. is included into Max.

this means, if ρ is a fun., of time, then it should be reflected in the Ampere's law

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$

•• Gauss Law $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ ρ

use continuity Equ. $\vec{\nabla} \cdot J = \frac{\partial}{\partial t} \left[\epsilon_0 \nabla \cdot \vec{E} \right]$

$$\vec{\nabla} \cdot \vec{J} = -\epsilon_0 \vec{\nabla} \cdot \frac{\partial}{\partial t}$$

$$\vec{\nabla} \cdot \left\{ \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right\} = 0$$

this term has the same unit of J

Title:

Therefor, in dynamics, we have two terms regarding current density

$$\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J} \longrightarrow \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

statics $\rightarrow$ dynamics, The Ampere's Law has to modified with this new $\vec{J}$, that is

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \left\{ \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right\}$$

$$= \mu_0 \vec{J} + \underbrace{\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}_{\text{Maxwell's Correction to Ampere law}}$$

Gauge transformation 10

Title: [redacted]

معنى gauge
تفاضل الثابت هو صفر معروف
ولكن هل أعلم بمعرفة هذا الثابت
هناك خيارات لا نهائية لذلك

$\frac{\partial}{\partial t}$ constant $= 0$

↓

infinite choices

So there is a rute in order to choose a constant ⇒ means fixing values ⇒ gauge transformation

in electrodynamics

↓

⇒ Coulombe gauge

⇒ Lorentz gauge

* Gauge transformation and gauges

consider

$$\int x \cdot dx = \frac{x^2}{2} + c \Rightarrow \frac{d}{dx}\left[\frac{x^2}{2} + c\right] = x$$

this is because $\frac{d}{dx} c = $ zero

Title:

So, we have the freedom in choosing the constant while maintaing

$$\frac{d}{dx}\left\{\frac{x^2}{2}+c\right\}=x$$

fixing the constant is called a gauge

Similarly: ~~variable~~ also

$$\int xy\,dx = \frac{y x^2}{2}+f(y)$$

because

$$\frac{\partial}{\partial x}\left\{\frac{y x^2}{2}+f(y)\right\}=xy$$

and

$$\frac{\partial}{\partial x}\left\{f(x)\right\}=0$$

therefore, we have the freedom to choose any function of y (not a constant)

fixing f(y) is a kind of gauge

Title:
In the gauge of electrostatics,

$\vec{\nabla} \times \vec{E} = 0$

Vector calculus: $\nabla \times (\vec{\nabla} \phi) = 0 \Rightarrow$ always

another rules

ϕ ↳ A scalar fun., $\phi(x,y,z)$

and $\vec{\nabla} C = 0$

↳ constant

This means

nothing but zero

$\vec{E} = \{ \vec{\nabla} \phi(x,y,z) + \vec{\nabla} C \}$ C: constant

$\vec{E} = -\vec{\nabla}(\phi + C)$? $\ominus$ electric potential at ∞ equal zero

⇒ If we choose/fix a constant c, then it is a gauge

$\phi = \phi(x,y,z)$

For magnetic field (static and dynamics)

$\vec{\nabla} \cdot \vec{B} = 0$

Vector Calculus: div (curl) $= 0 \Rightarrow \vec{B} = \vec{\nabla} \times \vec{A}$

$\Rightarrow \vec{\nabla} \cdot \{ \vec{\nabla} \wedge \vec{A} \} = 0$

↳ any vector

Then, the vector $\vec{A}$ is known magnetic vector potential

Title:

just a name not physical reasons

note: (A) magnetic vector potential is not potential energy

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$$

vector calculus: curl (grad) = 0

$$\boxed{\vec{\nabla} \times \{ \vec{\nabla} \phi \} = 0}$$

valid for any scalar fun, of x, y, z

used here

we can arrive $\vec{B} = \vec{\nabla} \times \{ \vec{A} + \vec{\nabla} g \}$

freedom to adding a scalar fun,

$$\vec{B} = \vec{\nabla} \times \{ \underbrace{\vec{A} + \vec{\nabla} g}_{\text{New magnetic vector potential}} \}$$

على اعتبار ان نضيف الـ $\vec{\nabla} g$, لكنها تعطي نفس الـ $\vec{B}$

By comparing with equ. $\vec{B} = \vec{\nabla} \times \vec{A}$

$\vec{B} = \vec{\nabla} \times \{ \vec{A} + \vec{\nabla} g \}$

can be represent by many infinite magnetic vector potential

Title:

Therefore $\vec{A}_{new} = \vec{A}_{old} + (\vec{\nabla} g)$

↓ give identically sam $\vec{B}$

this way we can generate many mag., vec., pot., for a given $\vec{B}$.

⇒ Hence we say that mag., vec., pot., has arbitrariness to the extent of gradient of a scalar

Consider, $\vec{A} = \vec{A}_0 + \vec{\nabla} g$ ↙ New $\vec{A}$ ↓ old $\vec{A}$ ↓ any scalar fun., of (x, y, z)

$$\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot \{ \vec{A}_0 + \vec{\nabla} g \}$$

$$\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot \vec{A}_0 + \nabla^2 g$$

we Known div(grad) $= \nabla^2$

Here A_0 and A, both represent same $\vec{B}$

Hence generally $\vec{\nabla} \cdot \vec{A}_0 \neq 0$ ⇒

~~lead~~ we have a lot more freedom to choose g

$\vec{B} = \vec{\nabla} \wedge \vec{A}_0$ ✓

Also

$\vec{B} = \vec{\nabla} \wedge A$ ✓

There is no reason to expect $\vec{\nabla} \cdot \vec{A}_0 = 0$

Title:

Now ~~...~~ we'll exploit this freedom such that $\vec{\nabla} \cdot \vec{A}_0 = -\nabla^2 g$

$\downarrow$

This means, we find / determine the fun., such that

$$\nabla^2 g = -\vec{\nabla} \cdot \vec{A}_0$$

successful $\Longrightarrow$ $\vec{\nabla} \cdot \vec{A} = 0$ it is easy to solve this Equation

gauge ده معناه ، عملنا تثبيت لتحويل
بدل ما كان $\underline{A}$ ليه اختيارات كتير طلع
$\vec{\nabla} \cdot \vec{A}_0 = -\nabla^2 g$ بشرط معين

Conclusion:

For a given mag., field $\vec{B}$, we found that mag., Vec., pot., $\vec{A}$ such that $\vec{\nabla} \cdot \vec{A} = 0 \Rightarrow$ This is ~~the~~ known as a coulomb gauge, because we have fixed the arbitrariness in $\vec{A}$.

Title:

Then, we say that we're working in the coulomb gauge

∘ Consequence: for the discovered $\vec{A}$, we still have $\vec{\nabla}\cdot\vec{B}=0$ and

$\vec{\nabla}\wedge\vec{B}=\mu_0\vec{J}$ (magnetostatics)

Since $\vec{B}=\vec{\nabla}\wedge\vec{A}$ we have

$$\vec{\nabla}\times\vec{B}=\vec{\nabla}\times\{\vec{\nabla}\times\vec{A}\}$$

and vector calculus curl curl = grad(div) − Lap

Therefore, $\vec{\nabla}\wedge\vec{B}=\vec{\nabla}\times\{\vec{\nabla}\times\vec{A}\}$

$$=\vec{\nabla}(\vec{\nabla}\cdot\vec{A})-\vec{\nabla}^2\vec{A}$$

under the coulombe gauge $\vec{\nabla}\cdot\vec{A}=0$, Hence

$\vec{\nabla}\times\vec{B}=-\vec{\nabla}^2\vec{A}$ and maxwell's eq.

$$\vec{\nabla}\wedge\vec{B}=\mu_0\vec{J}$$

Therefore

$$\nabla^2\vec{A}=-\mu\vec{J}$$

this is like poisson's equ.;

vector poisson equ., in vector form

3 differential p.p. equation

Title:

for comparison;

$\nabla^2 \phi = -\rho/\epsilon_0$ in electrostatics is the scalar equation