Linear Effects: Electron Plasma Waves

Waleed Moslem

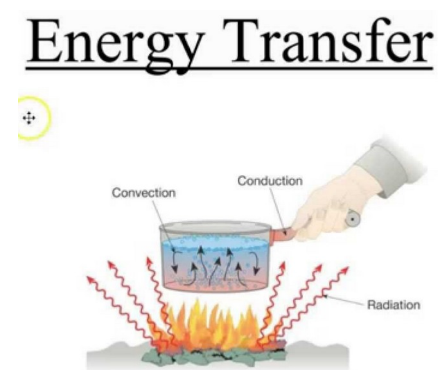
Aim of the lecture

- Why waves?
- Types of plasma waves
- Electron plasma waves characteristics
- Mathematical description
- Landau damping
- Applications of electron plasma waves

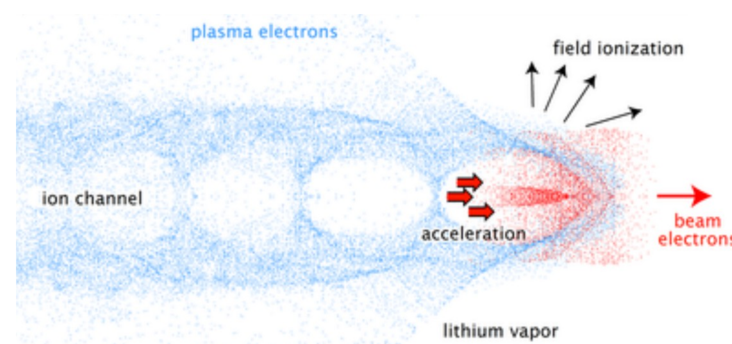
Why waves?

What is the importance to study waves in plasma?

(a) Plasma fingerprints appear in wave emissions. Thus, they are useful in faraway or unavailable plasma observation. They can serve as diagnostic tools.



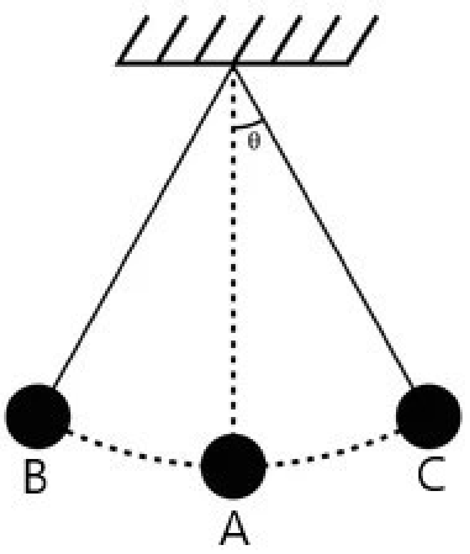
(b) Plasma waves are essential for several processes, including energy transfer, ionospheric loss, particle acceleration, and heating.



Types of plasma waves

What are Plasma Waves?

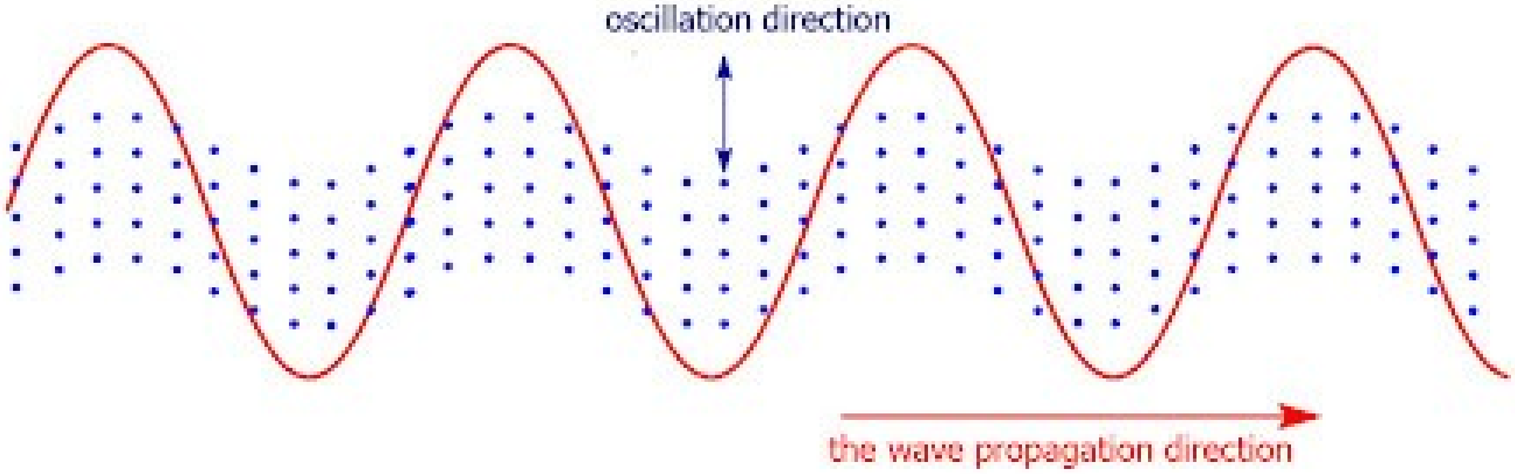
- Frequency & Waves
- Plasma waves are oscillations in the density of a plasma
- Electrons and ions oscillate around their equilibrium positions



Types of plasma waves

What are Plasma Waves?

- **Frequency & Waves**
- **Plasma waves are oscillations in the density of a plasma**
- **Electrons and ions oscillate around their equilibrium positions**



Types of plasma waves

Electron waves (electrostatic)

$\mathbf{B}_0 = 0$ or $\mathbf{k} \parallel \mathbf{B}_0$:	$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{th}^2$	(Plasma oscillations)
$\mathbf{k} \perp \mathbf{B}_0$:	$\omega^2 = \omega_p^2 + \omega_c^2 = \omega_h^2$	(Upper hybrid oscillations)

Types of plasma waves

Electron waves (electromagnetic)

$\mathbf{B}_0 = 0$: $$\omega^2 = \omega_p^2 + k^2c^2$$ (Light waves)

$\mathbf{k} \perp \mathbf{B}_0, \mathbf{E}_1 \parallel \mathbf{B}_0$: $$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$ (O wave)

$\mathbf{k} \perp \mathbf{B}_0, \mathbf{E}_1 \perp \mathbf{B}_0$: $$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2}$$ (X wave)

$\mathbf{k} \parallel \mathbf{B}_0$: $$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - (\omega_c/\omega)}$$ (R wave)
(whistler mode)

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 + (\omega_c/\omega)}$$ (L wave)

Electron plasma waves characteristics

Definition

Electron Plasma Waves are longitudinal oscillations of the electron component of a plasma

Also known as Langmuir waves

Notes:

- High frequency, why?
- Independent of ion motion, why?

Irving Langmuir
USA
(1881 – 1957)
Nobel Prize in Chemistry 1932

Electron plasma waves characteristics

Frequency Range

Electron plasma waves typically occur at high frequencies, on the order of the plasma frequency (approx. in MHz-GHz).

Wavelength

These waves have short wavelengths compared to ion plasma waves due to the lighter mass and higher mobility of electrons.

Dispersion Relation

$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{\text{th}}^2$$

Electron plasma waves characteristics

Polarization

Electron plasma waves are longitudinal, meaning the oscillations of electrons are parallel to the direction of wave propagation.

Energy Transport

These waves carry energy through the plasma via the oscillatory motion of electrons.

Mathematical description

$$mn_e\left[\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla)\mathbf{v}_e\right] = -en_e\mathbf{E}$$

$$F = m\frac{d^2x}{dt^2} \qquad F = -eE$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon_0}$$

Mathematical description

$$\rho(x) = ne \qquad \frac{dE}{dx} = \frac{\rho(x)}{\epsilon_0} \qquad E = \left(\frac{ne}{\epsilon_0}\right)x$$

The electric field increases linearly with distance from the origin due to a uniform charge density.

$$m\frac{d^2x}{dt^2} = -e\left(\frac{ne}{\epsilon_0}\right)x$$

$$\frac{d^2x}{dt^2} + \left(\frac{ne^2}{\epsilon_0 m}\right)x = 0$$

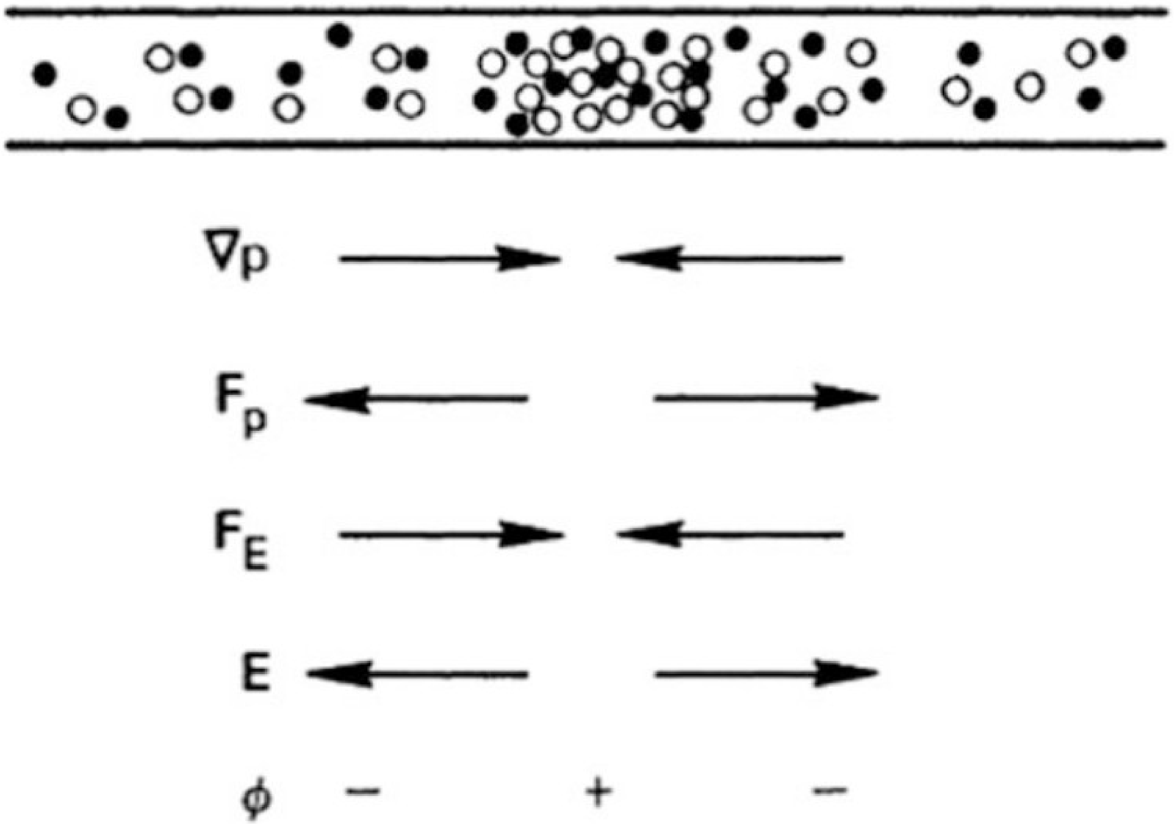
$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \omega = \sqrt{\frac{ne^2}{\epsilon_0 m}}$$

Forces in Plasma

- Inertial force
- Electric force
- Magnetic force
- Pressure gradient force
- Collisional force
- Drag force
- Corilis force
- Ponderomotive force
- Viscosity
- Tunnling force
- Exchange-correlation force
- Gravitational force
- Thermophoretic force
- Radiation pressure force
- Diffusion force
- **15 Forces**

Pressure Gradient Force

- **Definition**: A difference in pressure inside a plasma creates the pressure gradient force, which moves particles from high-pressure regions to low-pressure regions.



$$\mathbf{F}_{pg} = -\nabla P$$

Mathematical description

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

$$mn_e \left[\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla) \mathbf{v}_e \right] = -en_e \mathbf{E} - \nabla p_e$$

$$\nabla p_e = 3KT_e \nabla n_e = 3KT_e \nabla (n_0 + n_1) = 3KT_e \frac{\partial n_1}{\partial x} \hat{\mathbf{x}}$$

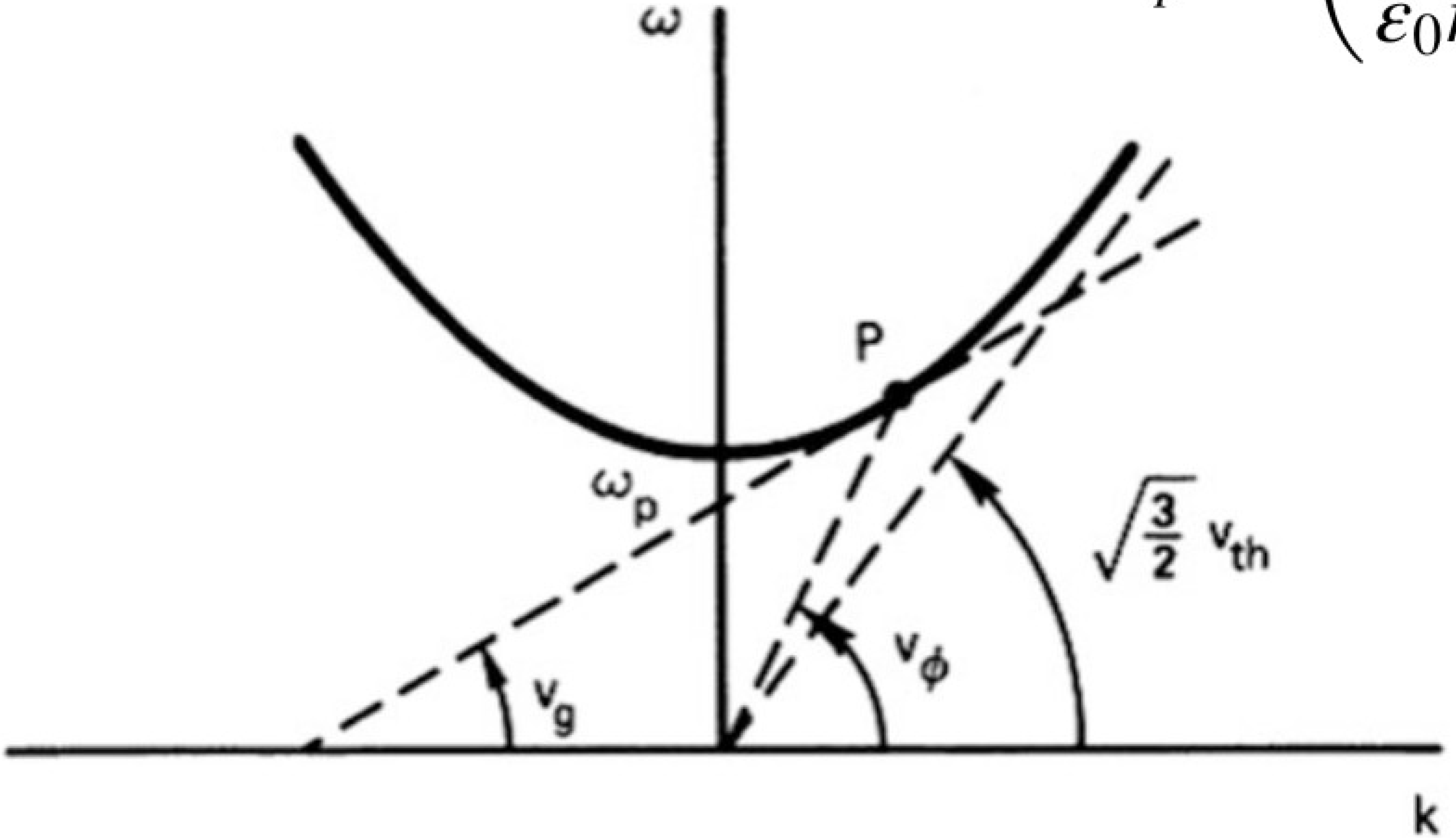
$$mn_0 \frac{\partial v_1}{\partial t} = -en_0 E_1 - 3KT_e \frac{\partial n_1}{\partial x} \qquad \varepsilon_0 \nabla \cdot \mathbf{E}_1 = -en_1$$

Mathematical description

$$v_{\text{th}}^2 \equiv 2KT_e/m$$

$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{\text{th}}^2$$

$$\omega_p = \left(\frac{n_0 e^2}{\varepsilon_0 m}\right)^{1/2}$$



$$\omega^2 = \omega_p^2 + \frac{3}{2} k^2 V_{th}^2 \quad (1)$$

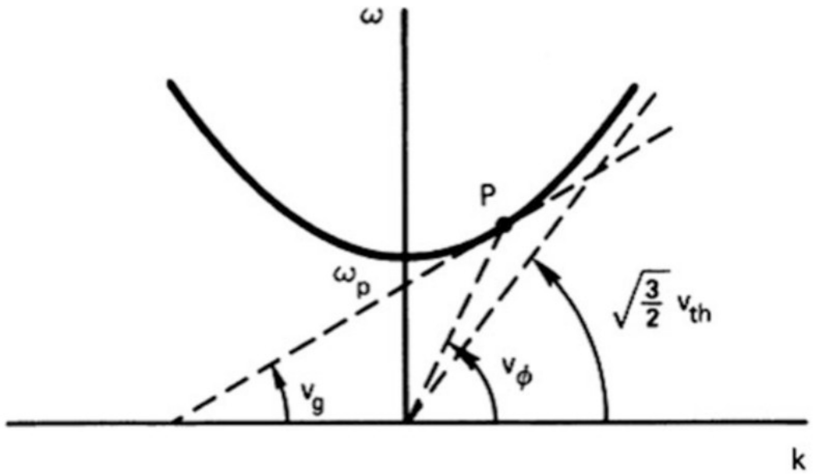
$$\frac{\omega^2}{k^2} = \frac{\omega_p^2}{k^2} + \frac{3}{2} V_{th}^2$$

$$= \frac{3}{2} V_{th}^2 \left[1 + \frac{2}{3 V_{th}^2} \frac{\omega_p^2}{k^2} \right]$$

$$\frac{\omega}{k} = \sqrt{\frac{3}{2} V_{th}^2} \left[1 + \frac{2}{3 V_{th}^2} \frac{\omega_p^2}{k^2} \right]^{1/2}$$

at $k \to \infty$

$$\frac{\omega}{k} = V_\phi = \sqrt{\frac{3}{2}} V_{th}$$



From Eq (1)

$$2\omega \, d\omega = \frac{3}{2} V_{th}^2 \, 2k \, dk$$

$$V_g = \frac{d\omega}{dk} = \frac{3}{2} V_{th}^2 \frac{k}{\omega} = \frac{3}{2} V_{th}^2 / V_\phi$$

Landau damping

Definition:

A mechanism by which the amplitude of plasma waves decreases over time due to resonant interactions between the wave and electrons moving at the phase velocity of the wave.

Importance:

Explains why certain plasma waves naturally attenuate, even in the absence of collisions.

Landau damping

Fundamental Concept

- Electrons moving at speeds close to the wave's phase speed (Vp) can either gain or lose energy to the wave.
- Electrons with velocities below Vp can gain energy from the wave, dampening it.
- Electrons with velocities slightly above Vp can decelerate and transfer energy to the wave.
- Net damping occurs when more electrons gain energy than they lose.

Landau damping

Applications of electron plasma waves

Plasma Diagnostics

Measuring plasma density and temperature.

Fusion Research

Controlling instabilities in fusion plasmas.

Space Physics

Understanding space weather phenomena (Solar Flares, Geomagnetic Storms, Auroras, ...etc).

Astrophysical Plasmas

Observed in solar wind, interstellar medium, and planetary magnetospheres.

How many forces we have used?

- Inertial force
- Electric force
- Magnetic force
- Pressure gradient force
- Collisional force, expect?
- Drag force
- Corilis force
- Ponderomotive force
- Viscosity
- Tunnling force
- Exchange-correlation force
- Gravitational force
- Thermophoretic force
- Radiation pressure force
- Diffusion force
- **15 Forces**

Summary

Terminology	Concept
Plasma waves	????
Electrostatic waves	????
Electromagnetic waves	????
Langmuir wave	????
Leading forces	????
Landau damping	????
Resonant interactions	????
Linear effects	????