Waves: Terminology & Concepts & principles

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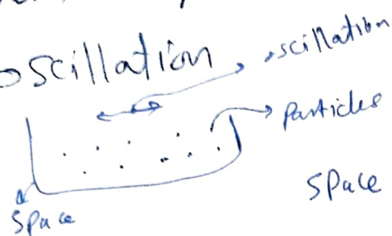
Physics

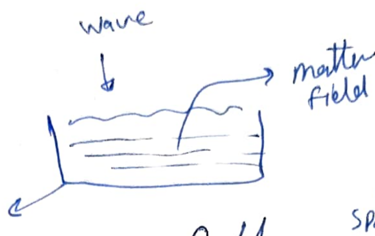
Particle

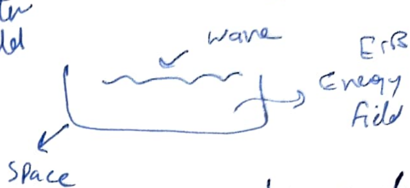
- Newton's eq
- oscillation

Field

- Wave equation.
- Wave.







- Physical system: particle or field.
- perturbation: an external force that not included in the system.
- response: ① shielding ② oscillation ③ wave.
- Wave: A disturbance in the field that propagates through the medium, carrying momentum and energy not matter.

due to collective behaviour of medium

* Equilibrium: ① static ② Dynamic

- System response: ① shielding: the system overcome the perturbation $\omega < \omega_p$ and the if the perturbation energy $E_{per} < E_{Inertia}$ and the perturbation dies out - like Debye shielding.

② oscillation: if the perturbation energy is much greater then the inertia energy i.e $\omega \gg \omega_p$, the system will not react back, so it will be controlled totally by the perturbation.

③ Wave: if the ~~system~~ perturbation energy comparable to the inertia energy $\omega \simeq \omega_p$, the system will oscillate and drift

wave: oscillation + drift

①

* classification of waves: ~~Based~~ on

- Based on:

① medium:
- mechanical : needs medium
- Electromagnetic : No medium

② ~~magnetic~~ Orientation to propagation:
- longitudinal : $\vec{u}_1 \parallel \vec{k}$, $\vec{k} \parallel \vec{E}$
- transverse : $\vec{u}_1 \perp k$. $\vec{k} \perp \vec{E}$

③ orientation to magnetic field:
- Parallel : $\vec{k} \parallel \vec{B}$
- ~~perpendicular~~ : $\vec{k} \perp \vec{B}$
- oplique : $\vec{k} \wedge \vec{B}$

④ orientation to electric field:
- Electrostatic : $\vec{k} \parallel \vec{E}$: $\vec{k} \times \vec{E} = 0 \rightarrow \nabla \times E \rightarrow \frac{\partial B}{\partial t} = 0$
- Electromagnetic : $\vec{k} \perp \vec{E}$: $\vec{k} \times \vec{E} \neq 0 \rightarrow \vec{\nabla} \times \vec{E} \neq 0 \rightarrow \frac{\partial B}{\partial t} \neq 0$

⑤ Speed:
- standing : $v = 0$
- travelling : $v \neq 0$

⑥ time:
- pulse : $f(t) \neq f(t+T)$
- Periodic : $f(t+T) = f(t)$

⑦ Wavefront : ① plane wave
② spherical wave
③ cylindrical wave

General wave equation:

- The general wave equation:

$$\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = H(x,t)$$ ← Response of the medium (particles/field) on the wave.

$\frac{1}{v^2}$ ← natural response of the medium

$V(x,t)$: is a function of properties of medium

(i) if $H(x,t) = 0$ homogenous static $V = \text{const}$

(ii) $H(x,t) \neq 0 \to V \neq \text{const}$

(i) Homogenous wave equation: $H(x,t) = 0$

- if the medium is static, uniform, linear, non-dispersive $\}$. $H(x,t) = 0$

No feedback from the medium

So,

$$\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0$$ ← No feedback from the medium

(1) in homogenous medium: $H = H(x) \to V = V(x)$

(2) Dynamic medium $H = H(t) \to V = V(t)$

(3) Dynamic / inhomogenous medium $H = H(x,t) \to V = V(x,t)$.

$$\Rightarrow f(x,t) = \underbrace{f(x-vt)}_{\text{travell to right}} + \underbrace{g(x+vt)}_{\text{travell to left.}}$$

$\Rightarrow f(x,t)$: the disturbance in the field. Propagate with V.

$\Rightarrow V$: Contains
- types of perturbation → restoring force
- reaction of active medium → Inertia

Exp:

1- Electromagnetic in vacuum:

$$V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ → Inertia, Perturbation

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = 0$$

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0$$

$$\nabla^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = 0$$

2- Sound in air:

$$V = \sqrt{\frac{P}{\rho}}$$ ← Perturbation type (P), Inertia (ρ)

3- Wave in string: $V = \sqrt{\frac{T}{\mu}}$ ← Perturbation (T) → Inertia (μ)

4- Sound in fluid: $V = \sqrt{\frac{B}{\rho}}$ → P, → I

5- In electric circuit ⇒ $V = \sqrt{\frac{1}{LC}}$.

(11) Nonhomogenous wave equation:

$\mathcal{H}(x,t)=0$
- → Nonlinear medium,
- → Dynamic medium.
- → Non-uniform " .
- → Dispersive " .

Exp: electromagnetic wave in Conducting medium. → feedback from the medium on the field or wave.

1- Electric field:

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial \vec{J}(\vec{r},t)}{\partial t} + \frac{1}{\epsilon_0}\vec{\nabla}\rho(\vec{r},t)$$

where $\vec{H}(x,t) = \mu_0 \frac{\partial \vec{J}(\vec{r},t)}{\partial t} + \frac{1}{\epsilon_0}\vec{\nabla}\rho(\vec{x},t)$

$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial A}{\partial t}$

2- Electrostatic Potential.

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{1}{\epsilon_0}\rho(\vec{r},t).$$

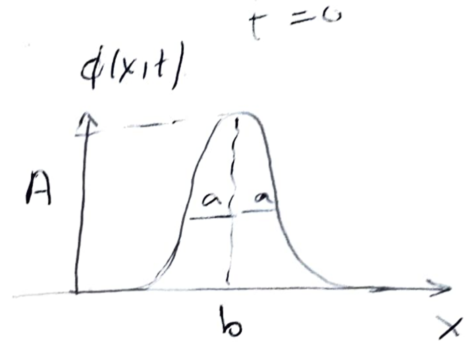
3- magnetic potential

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\mu_0 J(\vec{r},t).$$

Gaussian Pulse:

- The general form of the Gaussian Pulse:

$$\phi(x,t) = A\, e^{-\left[\frac{(x-b\mp vt)^2}{a^2}\right]}$$



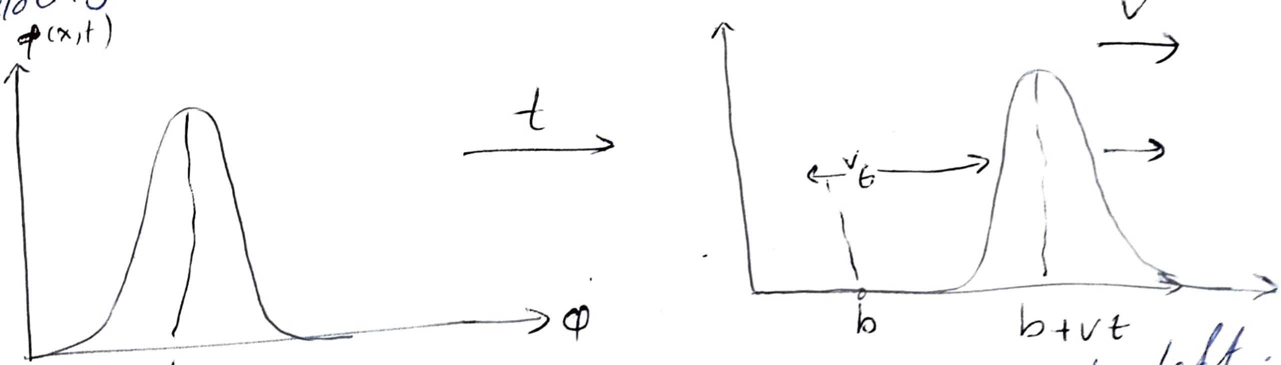
(i) for $t=0$

$$\phi(x,t) = A\, e^{-\frac{(x-b)^2}{a^2}}$$

- A: amplitude.
- b: the drift position of the peak.
- a: half the width.
- $2a$: the full width.

(ii) for $t \neq 0$:

- The pulse holds its shape and travel to right with definite velocity v.

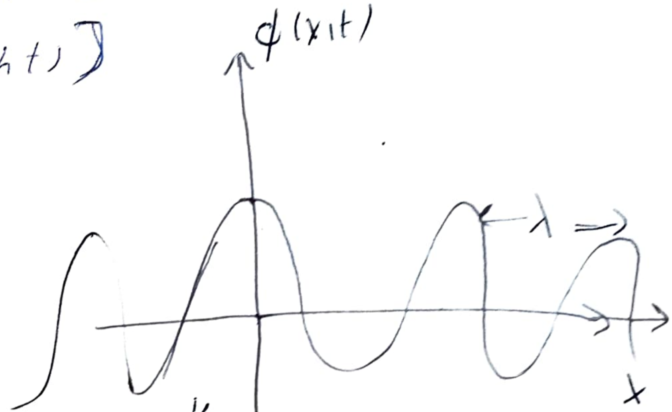


Note that: if $v<0$ or $(+v)$: the pulse travel to the left.

* Sinusoidal ~~wave~~ Single wave:

- the travelling Sinusoidal wave:

$$\phi(x,t) = A \cos(kx - \omega t) \text{ ; "angular form"}$$
$$= A \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] \text{ ; "linear form"}$$
$$= A \cos[k(x - v_{ph} t)]$$



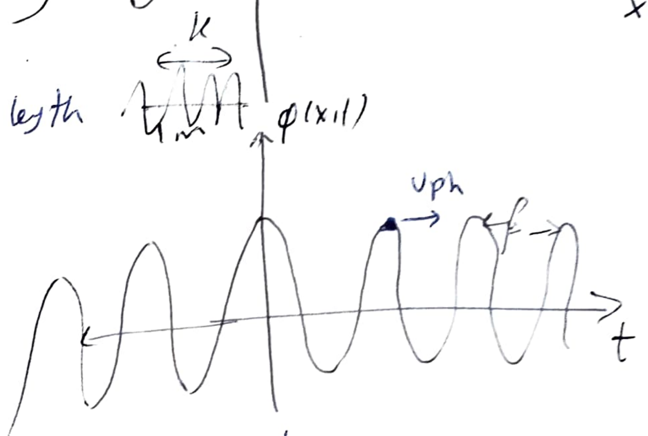
⇒ A: maximum amplitude (m)

⇒ λ: Wavelength (m)

⇒ f: linear frequency (HZ).

⇒ $k = \frac{2\pi}{\lambda} \; \frac{rad}{m}$:

- Wavenumber: no of waves per unit length
- Spatial frequency.
- angular wavelength



⇒ $\omega = \frac{2\pi}{\cancel{p}} f$: $\frac{rad}{sec}$

- angular frequency.
- temporal frequency: no of waves per second.

⇒ $v_{ph} = \frac{\omega}{k}$; phase $= kx - \omega t$.

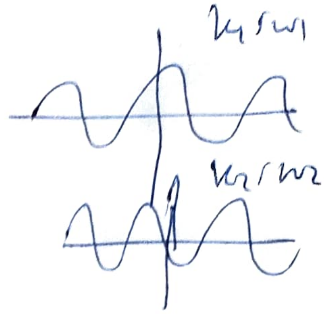
$(kx - \omega t) = Const$

$\frac{d}{dt}(kx - \omega t) = Const \Rightarrow \boxed{\frac{dx}{dt} = \frac{\omega}{k} = v_{ph}}$

v_{ph}: the speed of a constant phase.

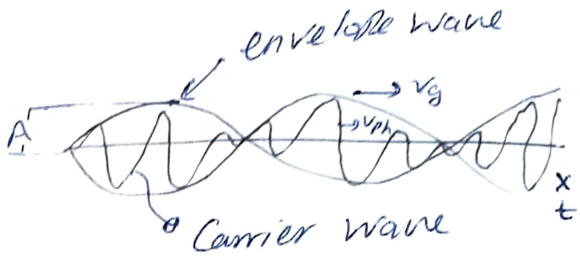
Sinusoidal Wavepacket:

- the wavepacket from two waves:



$$\phi(x,t) = A \cos[\Delta k x - \Delta w t] \cos[k_0 x - w_0 t]$$

where A: amplitude of the envelope.



$\cancel{\cos(\Delta k x - \Delta w t)}$

(i) Carrier wave:

⇒ $\cos(k_0 x - w_0 t)$,

⇒ $k_0 = \dfrac{k_1 + k_2}{2}$; $w_0 = w_1 + w_2$

⇒ $v_{ph} = \dfrac{w_0}{k_0}$: phase speed

⇒ fast scale wave → fast variation in space
→ " " " time

(ii) Envelope wave:

⇒ $\cos(\Delta k x - \Delta w t)$

⇒ $\Delta k = \dfrac{k_2 - k_1}{2}$; $\Delta w = \dfrac{w_2 - w_1}{2}$

⇒ $v_g = \dfrac{\Delta w}{\Delta k}$: $\dfrac{d}{dt}(\Delta k x - \Delta w t) = 0 \Rightarrow \dfrac{dx}{dt} = v_g = \dfrac{\Delta w}{\Delta k}$

if $v_g = \lim_{\substack{\Delta k \to 0 \\ \Delta w \to 0}} \dfrac{\Delta w}{\Delta k} = \dfrac{dw}{dk}$: group speed [speed of constant phase on the envelope]

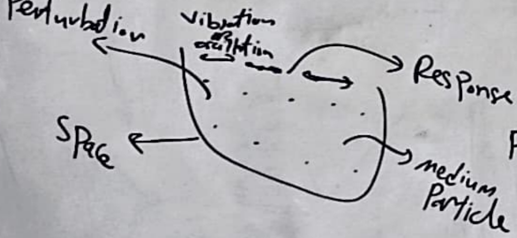
⇒ slow scale wave → slow spatial variation.
→ " temporal "

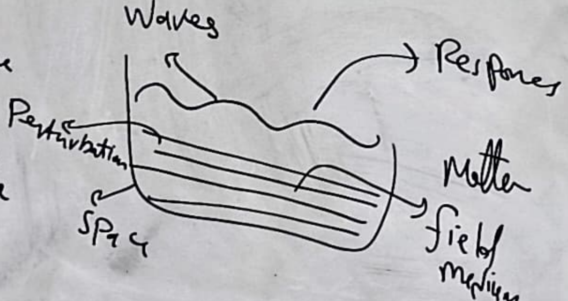
Classical Physics

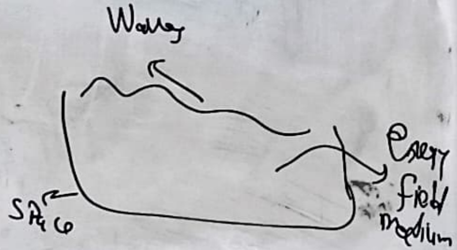
Particle theory

Field theory

* Physical system:







* Recipe:

(i) Equilibrium:

(ii) Perturbation/disturbance vs external force: (i) type (ii) strength.

(iii) Response: (i) shelding (ii) Vibration/oscillation (iii) Waves,

(iv) Study response: governing equation.

* Classification of wares:

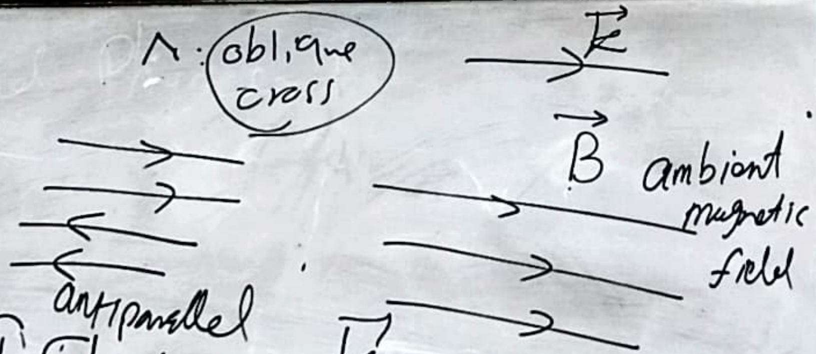
- Based on

1. Medium: (1) Mechanical (med) (2) Electromagnetic (no medium)
2. Orientation to propagation: (1) Longitudinal // : Perturbation // Propagation.
   (2) transverse : Perb $\perp$ Prop.

*Classification of waves:

- Based on

③ orientation to electric field:



① Electrostatic: Propagation // Electric field. $\vec{K} // \vec{E}$

② Electromagnetic: $\vec{K} \perp \vec{E}$.

orientation to magnetic field:

① Parallel: $\vec{K} // \vec{B}$

② Perpendicular: $\vec{K} \perp \vec{B}$

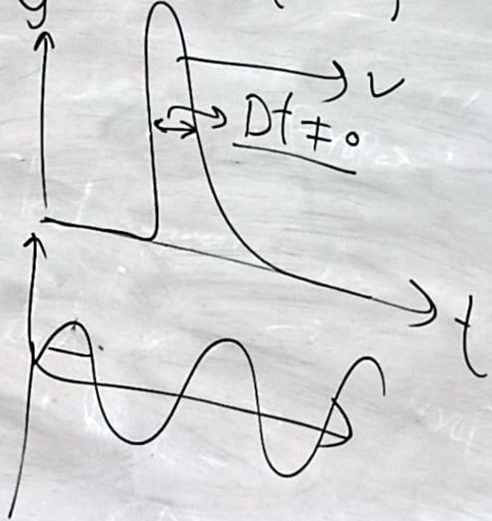
③ oblique: $\vec{K} \wedge \vec{B}$

* Classification of waves:

5 Speed: 1 travelling ($v \neq 0$) 2 standing. ($v = 0$)

6 time duration: 1 Pulse

2 Periodic:



* the general wave equation:

1D: $$\frac{\partial^2 f(x,t)}{\partial x^2}\bigg|_{t=t_0} - \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}\bigg|_{t=t_0} = F(x,t)$$ ← inhomogenous term

$v(x,t)$

$v^2 = \frac{T}{\mu}$

(i) Static-homogenous: $F(x,t) = 0$

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0$$

$v = \text{Const}$.

(i) EMW in vacuum: $\frac{\partial^2 E}{\partial x^2} - \frac{1}{\mu_0 \epsilon_0}\frac{\partial^2 E}{\partial t^2} = 0$, $v = c$. (B)

(ii) Sound in air: $\frac{\partial^2 \rho}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \rho}{\partial t^2} = 0$, $v^2 = \frac{P}{\rho}$ ← Inertia

(iii) Wave in string: $\frac{\partial^2 H}{\partial x^2} - \frac{1}{v^2} \dots = \dots$

* Sinusoidal wave:- Liner frequency

λ: Lambda

λ: m , f: sec^{-1}

$$E(x,t) = E_0 \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right]$$ Linear form

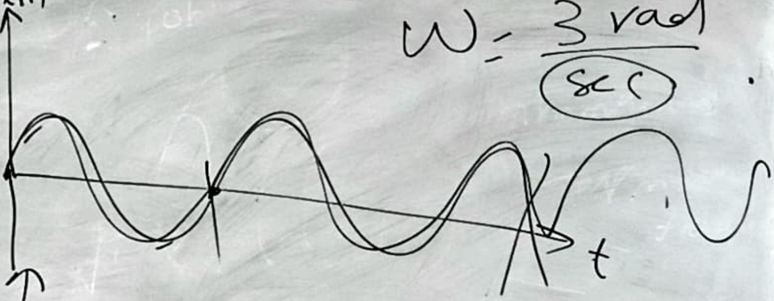
$$= E_0 \cos(kx - \omega t)$$: angular form

Wave number ← K: Spatial frequency

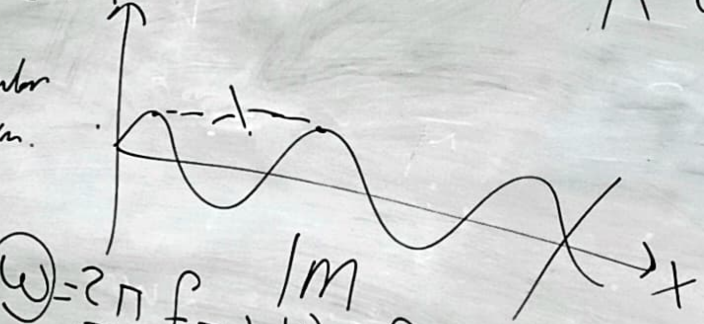
$= \frac{rad}{m}$

angular freq ← ω: temporal frequency

$= \frac{rad}{sec}$



$$\omega = \frac{3\ rad}{sec}$$



1m

$\omega = 2\pi f \rightarrow \frac{\omega}{2\pi} = f$

$k = \frac{2\pi}{\lambda} \rightarrow \frac{k}{2\pi} = \frac{1}{\lambda}$