د. عبدالعظيم القحاشي

Single particle model (SPM) "Solid state"
Particle orbit theory (POT)

- the dynamic behaviour of a plasma is governed by the interaction between the plasma particles and the internal fields produced by the particle themselves, and externally applied fields.

*** plasma models:**

i- Single particle model: $L < \lambda_D$ — microscopic (classical mechanics theory + EMT)

ii- Kinetic model: $L > \lambda_D$ — mesoscopic (statistical mechanics theory + EMT)

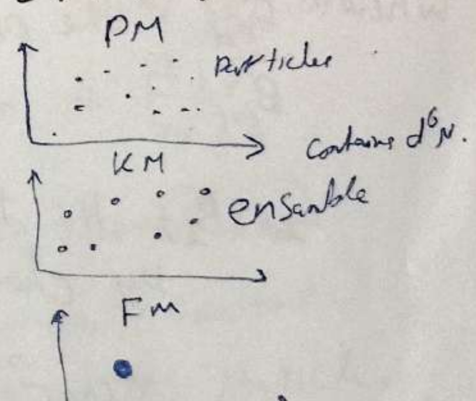
iii- Fluid model: $L >> \lambda_D$ — macroscopic (Fluid mechanics theory + EMT)

*** Coordinates:**

i- Cartesian: 3 lengths: x, y, z.

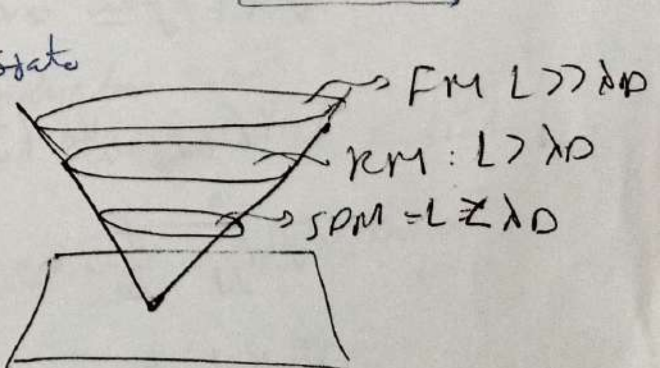
ii- cylindrical: 2 length + 1 angle: ρ, ϕ, z.

iii- spherical: 1 length + 2 angles: r, θ, ϕ.



* model

- Particle
  - Solid state discrete
  - Particle aspect
  - micro
  - CMT
  - Individual particles
  - Configuration
- kinetic
  - gas state ensamble
  - chunk aspect
  - meso
  - SMT
  - chunk
  - Phase
- fluid
  - liquid state
  - Continuum scale
  - macro
  - FMT
  - collective
  - Continuum
  - Physical space



1

* Single Particle model:

- It is useful for predicting the behaviour of very low densi[ty] plasma, such as Solar Corona, Cosmic rays, Van Allen radiation belt.

- the general formalism:

- the equation of motion of a charged particle of mass m, charge q, under the action of the Lorentz force F due to electric field (E) and magnetic induction (B) fields, can be written as

$$\frac{d\vec{P}}{dt} = \vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$$

$$= q\left[(\vec{E}_{ext} + \vec{E}_{Int}) + \vec{V} \times (\vec{B}_{ext} + \vec{B}_{In})\right]$$

where: E_{ext}: the prescribed and not affected by the charged particles.

B_{ext}: " " " " " " " .

E_{Int} & B_{Int}: the described by Maxwell's eqs and affected by charged particles.

$$\nabla \times \vec{E}_{Int} = -\frac{\partial \vec{B}_{Int}}{\partial t}$$

$$\nabla \times \vec{B}_{Int} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}_{Int}}{\partial t}\right)$$

$$\nabla \cdot \vec{E}_{Int} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot B_{Int} = 0$$

$$\rho = \sum_j n_j q_j \; ; \; \vec{J} = \sum_j n_j q_j \vec{v}_j$$

Terminology:

(i) time → static / Constant field: E & B are constant in time $\frac{\partial E}{\partial t} = \frac{\partial B}{\partial t} = 0$

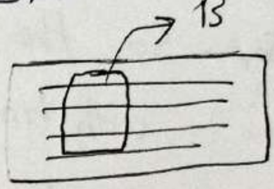
→ time-varying / Varying field: E(t) & B(t) depend on t. $\frac{\partial E}{\partial t} \neq 0, \frac{\partial B}{\partial t} \neq 0$

(ii) space → uniform: E & B don't depend on space. $\frac{\partial \vec{E}}{\partial \vec{r}} = 0$, $\frac{\partial \vec{B}}{\partial \vec{r}} = 0$

→ space-varying / nonuniform: " " depend on space. $\frac{\partial \vec{E}}{\partial \vec{r}} \neq 0, \frac{\partial \vec{B}}{\partial \vec{r}}$

Note that: - constant electric field → electrostatiatic.

- " magnetic " → magnetostatic.



*Energy conservation:

(i) Magnetostatic:

- In the absence of ($E = 0$) and B is constant.

$$m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}) \qquad W = \int \vec{F} \cdot d\vec{r}$$

$$= \int (\vec{v} \times \vec{B}) \cdot d\vec{r}$$

$$\vec{v} \cdot m \frac{d\vec{v}}{dt} = q\, \vec{v} \cdot (\vec{v} \times \vec{B}) = 0 \qquad \because \vec{F} \perp \vec{r}$$

$$\frac{d}{dt}\left(\frac{1}{2} m v^2\right) = 0 \quad ; \text{where } \vec{v} \cdot (\vec{v} \times \vec{B}) = 0 \qquad W = 0.$$

- This means that: - the particle's kinetic energy and the magnitude of its velocity (speed v) are both constant.
  - A static magnetic field does no work on the particle, since the magnetic force is perpendicular to $\vec{v}$.
  - A static magnetic field doesn't change the particle kinetic energy.
  - There is acceleration due to the rotation, i.e. direction of velocity.

(ii) magnetostatic and electrostatic:

$$m\frac{d\vec{V}}{dt} = q(\vec{E} + \vec{V}\times\vec{B})$$

no charge direction & charge only in magnitude zero

$$m\vec{V}\cdot\frac{d\vec{V}}{dt} = q(\vec{E}\cdot\vec{v}) + q\,\vec{V}\cdot(\vec{v}\times\vec{B}) \neq 0$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = q(\vec{E}\cdot\vec{v}) = P_{electric} \neq 0$$

$W = \int \vec{f}\cdot d\vec{r}$
$= q\int \vec{E}\cdot d\vec{r} \rightarrow W \neq 0$
$\vec{E} \parallel d\vec{r}$ charge

- So:- So, the particle kinetic energy and speed with time: $q>0$: increases $\parallel \vec{E}$; $q<0$: " antiparallel to $\vec{E}$.
- the electric field does work on the particle.
- " electrostatic change the particle kinetic energy.
- Where KE increases comes from?
- For magnetostatic: $\frac{\partial B}{\partial t} = 0$

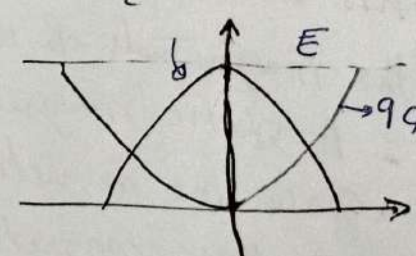
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = 0 \rightarrow \vec{\nabla}\times(\vec{\nabla}\phi) = 0$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$\therefore \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = -q(\vec{\nabla}\phi\cdot\vec{V}) = -q\,\vec{\nabla}\phi\cdot\frac{d\vec{r}}{dt} = -q\frac{d\phi}{d\vec{r}}\cdot\frac{d\vec{r}}{dt}$$

increase of KE due to Electric Potential energy

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = -q\frac{d\phi}{dt}$$



$$\frac{d}{dt}\left(\frac{1}{2}mv^2 + q\phi\right) = 0$$

$$\frac{d}{dt}(E) = 0$$

- But the total energy of the particle in electrostatic field is constant. $E = \frac{1}{2}mv^2 + q\phi$
$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \leftarrow$ Like spring

4

Electromagnetic: $\nabla\cdot\vec{B}=0 \rightarrow \nabla\times(\vec{\nabla}\times\vec{A})=0,\quad \vec{B}=\vec{\nabla}\times\vec{A}$

$$\vec{\nabla}\times\vec{E}+\frac{\partial B}{\partial t}=\vec{\nabla}\times\vec{E}+\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{A})$$

$$=\nabla\times\left(E+\frac{\partial A}{\partial t}\right)=0$$

$$=\nabla\times(-\vec{\nabla}\varphi)=0$$

$$\vec{E}+\frac{\partial\vec{A}}{\partial t}=-\vec{\nabla}\varphi$$

$$\boxed{\vec{E}=-\vec{\nabla}\varphi-\frac{\partial\vec{A}}{\partial t}}$$

$\overset{?}{\neq}0$

$$\therefore\ \frac{d}{dt}\left(\tfrac{1}{2}mv^2\right)=q(\vec{E}\cdot\vec{v})=q(-\vec{\nabla}\varphi\cdot\vec{v})+q\left(-\frac{\partial\vec{A}}{\partial t}\cdot\vec{v}\right)$$

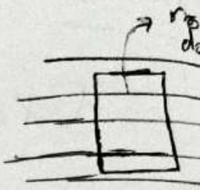
$$=-q\frac{d\varphi}{dt}-q\left(\frac{\partial\vec{A}}{\partial t}\cdot\vec{v}\right)$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2+q\varphi\right)=-q\left(\vec{v}\cdot\frac{\partial\vec{A}}{\partial t}\right).$$

- the system is not conserved and there is no energy integral.

* Charge Particle motion:

① ~~Constant~~ static and uniform Electromagnetic field.



② " and nonunifor " " "

③ ~~time-varying~~ inconstant or varying and uniform " " " "

④ time ~~-~~ varying and space- ~~varying~~ nonuniform electromagnetic field:

ⓘ charge particle motion in uniform electrostatic field:
↳ constant E.

$$\frac{d\vec{P}}{dt} = q\vec{E}$$

Drift — + : ‖ parallel — − : ∦ antiparallel

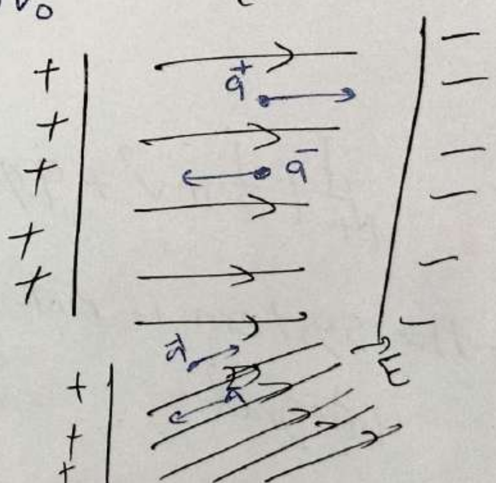
$$\vec{p}(t) = q\vec{E}t + P_0 = m\vec{v}$$

$$m\frac{d\vec{r}}{dt} = q\vec{E}t + P_0 = q\vec{E}t + mv_0$$

$$\frac{d\vec{r}}{dt} = \left(\frac{q\vec{E}}{m}\right)t + v_0$$

$$\boxed{\vec{r} = \frac{1}{2}\left(\frac{q\vec{E}}{m}\right)t^2 + \vec{v}_0 t + \vec{r}_0}$$

$$\boxed{\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}at^2}$$



- So, the particle moves with constant acceleration $\left(\frac{qE}{m}\right)$. ~~in~~

① $q>0$: in the direction of $\vec{F}$. (parallel) $\vec{a} \parallel \vec{E}$

② $q<0$: " " opposite " " " $\vec{E}$. (antiparallel. $\vec{a} \nparallel \vec{E}$

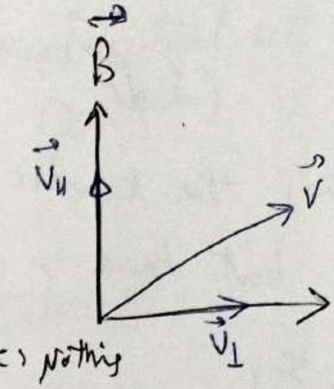
~~③~~ In perpendicular direction to the electric field, there is no acceleration and the particle state of motion remains unchanged ✓

↳ change in the state is only in parallel to $\vec{E}$.

Charged particle motion in uniform magnetostatic field:

the equation of motion: (Gyration).

$$m\frac{d\vec{V}}{dt} = q(\vec{V}\times\vec{B})$$



$$\vec{V} = \vec{V}_{\parallel} + \vec{V}_{\perp} \quad ; \quad \parallel \text{ Parallel to } \vec{B}. \quad \perp \text{ perp to } \vec{B}.$$

$$\frac{d\vec{V}_{\parallel}}{dt} + \frac{d\vec{V}_{\perp}}{dt} = \frac{q}{m}(\vec{V}_{\perp} * \vec{B}) + \cancel{\frac{q}{m}(\vec{V}_{\parallel} * \vec{B})} \quad \text{zero.}$$

(i) static: nothing

(ii) Parallel: linear motion

(iii) Perpendicular: circular motion linear or circular

(iv) oblique: L + C = Helical motion

$$\frac{d\vec{V}_{\parallel}}{dt} + \frac{d\vec{V}_{\perp}}{dt} = \frac{q}{m}(\vec{V}_{\perp} * \vec{B})$$

- Since the term $(\vec{V}_{\perp}\times\vec{B})$ is perpendicular to $\vec{B}$, So

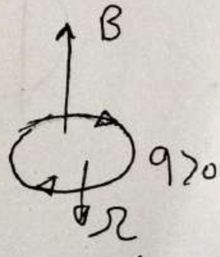
$$\frac{d\vec{V}_{\parallel}}{dt} = 0 \rightarrow \vec{V}_{\parallel} = \vec{V}_{\parallel 0} \text{ (initial velocity)} \quad L + C = H$$

& linear motion without acceleration

- the perpendicular component equation as:

$$\frac{d\vec{V}_{\perp}}{dt} = \frac{q}{m}(\vec{V}_{\perp}\times\vec{B}) \quad ; \quad \vec{B} = B\hat{B}.$$

$$= \frac{qB}{m}(\vec{V}_{\perp}\times\hat{B}) = -\frac{qB}{m}(\hat{B}\times\vec{V}_{\perp})$$



$$\frac{d\vec{V}_{\perp}}{dt} = (\vec{\Omega}_c \times \vec{V}_{\perp})$$

$$\boxed{\vec{\Omega}_c = -\frac{q\vec{B}}{m}} = -\Omega_c\hat{B} \quad ; \quad \Omega_c = \frac{|q|B}{m}$$

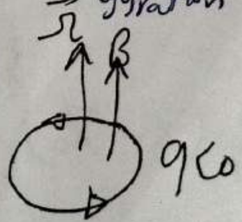
Ω_c:
- angular frequency of gyration
- gyrofrequency.
- Cyclotron frequency
- Larmor frequency.

* Notes:

① $q>0$: $\vec{\Omega}_c$ is antiparallel of $\vec{B}$ # $\uparrow\uparrow\uparrow\downarrow \vec{\Omega}_c$ → Clockwise direction gyration

② $q<0$: $\vec{\Omega}_c$ " parallel $\vec{B}$ $\parallel$. $\uparrow\uparrow\uparrow$ B $\uparrow\vec{\Omega}_c$. ↳ Counterclockwise

③ $\Omega_c \propto B$. Ω_c is constant as long as B is constant and uniform.
$\propto \frac{1}{m}$.



- Since Ω_c is constant, so

$$\frac{d}{dt}\vec{v}_\perp = -\vec{\Omega}_c \times \vec{v}_\perp \neq 0$$

$$\vec{v}_\perp \cdot \frac{d\vec{v}_\perp}{dt} = (\vec{\Omega}_c \times \vec{v}_\perp)\cdot\vec{v}_\perp = 0$$

$$\frac{dv_x}{dt} = \Omega_c v_y \qquad \frac{dv_y}{dt} = -\Omega_c v_x \qquad \frac{d^2 v_x}{dt^2} + \Omega_c^2 v_x = 0$$

$$v_x = v_\perp \sin(\Omega_c t + \theta_0) \qquad v_y = v_\perp \cos(\Omega_c t + \theta_0)$$

$\frac{dv_z}{dt} = 0$ $v_\perp$ doesn't change.

So, the kinetic energy and the speed $\frac{dv_\perp}{dt} = 0$ due to the change in the direction.
but there is acceleration
angular acceleration

- So,

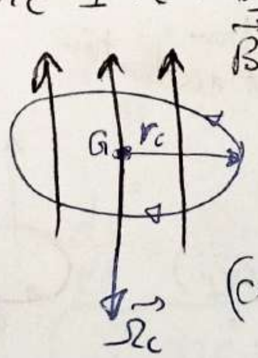
$$\frac{d}{dt}\vec{v}_\perp = -\vec{\Omega}_c \times \frac{d\vec{r}_c}{dt} = \frac{d}{dt}(-\vec{\Omega}_c \times \vec{r}_c). \quad r_c:\text{ centre of gyration}$$

guiding center.

$$\boxed{\vec{v}_\perp = -\vec{\Omega}_c \times \vec{r}_c} \rightarrow \boxed{r_c = \frac{v_\perp}{\Omega_c} = \frac{m v_\perp}{|q|B}}$$

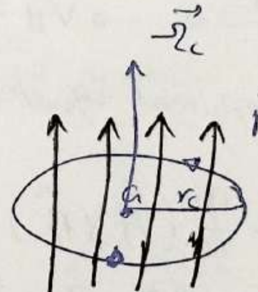
radius of gyration
gyroradius
cycltron radius
larmor radius.

So, $\vec{\Omega}_c \perp \vec{r}_c, \vec{v}_\perp$ $\quad v = r\omega$



(clockwise)

$q>0$



(counter clockwise)

$q<0$

$\vec{\Omega}_c \parallel \vec{L}$ (angular momentum)

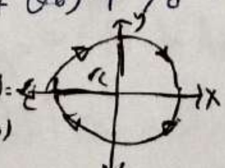
-* In Cartesian coordinates, the coordinates: $\quad$ r_c: same as long as
$\vec{B} = B_0\hat{z}$, $\vec{v}_\perp = v_x\hat{x} + v_y\hat{y}$. B and $v_\perp$ are constant uniform.

Ions

$$x(t) = -\frac{v_\perp}{\Omega_c}\cos(\Omega_c t + \theta_0) + X_0$$

$$y(t) = \underbrace{\frac{v_\perp}{\Omega_c}}_{r_c}\sin(\Omega_c t + \theta_0) + Y_0.$$

$$z(t) = v_\parallel t + z_0.$$

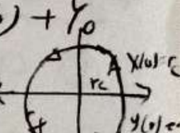


electrons

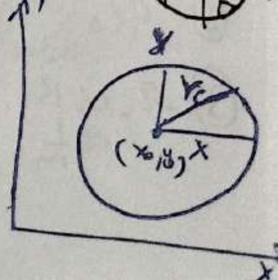
$$x(t) = \frac{v_\perp}{\Omega_c}\cos(\Omega_c t + \theta_0) + X_0$$

$$y(t) = \frac{v_\perp}{\Omega_c}\sin(\Omega_c t + \theta_0) + Y_0$$

$$z(t) = v_\parallel t + z_0$$



$$(X - X_0)^2 + (Y - Y_0)^2 = \left(\frac{v_\perp}{\Omega_c}\right)^2 = r_c^2$$



The resultant trajectory:

translation
- Superposition of : (i) uniform motion along B with constant $V_{\parallel}$.

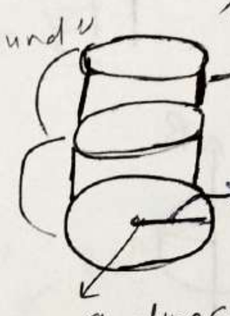
(ii) circular motion in the plane normal to B with constant speed $V_{\perp}$ (gyration)

= Helix (cylindrical).

constant
* Helix: ~~gyration~~ = circulation + translation (along $\vec{B}$).

① translation motion of guiding center along B.

② circular motion around " " in plane normal to B



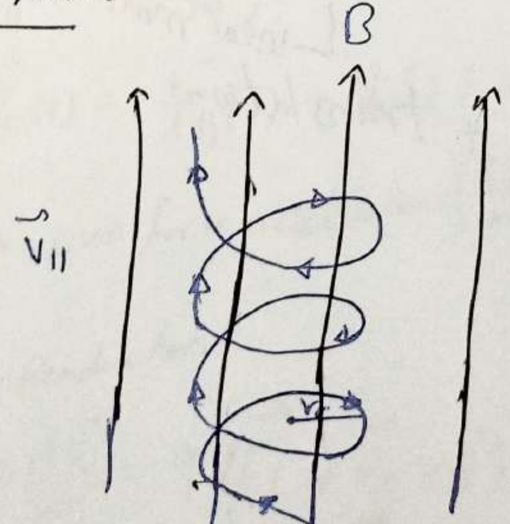
Constant pitch due to constant ($V_{\parallel}$) translation.

r_c: constant radius due to the constant $V_{\perp}$ & r_c.

|| linear, ⊥ circular, Helical

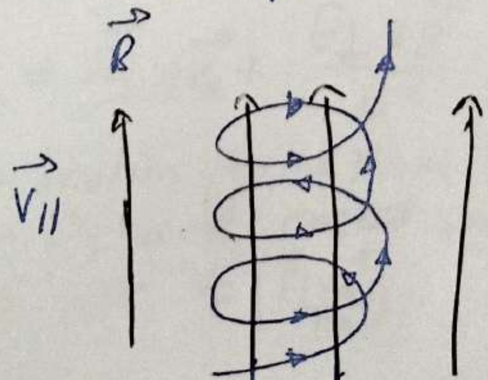
L + C = H

① Positive ions:



q > 0
(Clockwise)

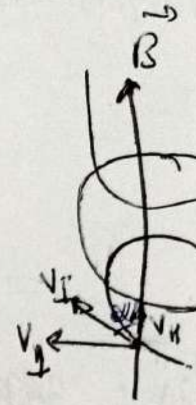
② electrons



q < 0
Counterclockwise

* Pitch angle:

- angle between $\vec{B}$ and the direction of motion:

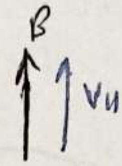


$$\alpha = \tan^{-1}\left(\frac{V_\perp}{V_\parallel}\right)$$

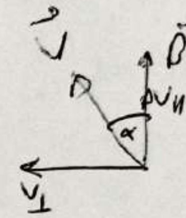
* Special cases:

(i) $V_\perp = 0$ & $V_\parallel \neq 0$, $\alpha = 0$, $r_c = 0$

uniform translation.



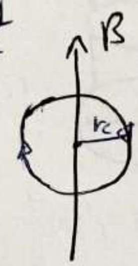
(ii) $V_\parallel = 0$ ← no initial velocity. & $V_\perp \neq 0$. $\alpha = \frac{\pi}{2}$



circular motion

gyration

rotation



(iii) $V_\parallel \neq 0$ & $V_\perp \neq 0$ Linear motion

Helix motion

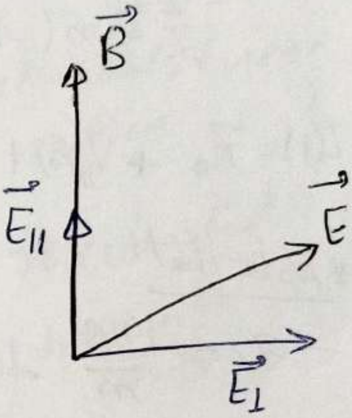
~~gyration~~ = circulation + translation

Charged particle motion in uniform and constant electromagnetic field

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Taking components parallel and perpendicular to B:

$$\vec{V} = \vec{V}_{\parallel} + \vec{V}_{\perp}$$
$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp}$$



∴ then: Parallel:

linear acceleration $\leftarrow m\frac{dv_{\parallel}}{dt} = qE_{\parallel}$ linear acceleration

$$\vec{V}_{\parallel}(t) = \left(\frac{q\vec{E}_{\parallel}}{m}\right)t + V_{\parallel}(0)$$

$$\vec{r}_{\parallel}(t) = r_{\parallel}(0) + V_{\parallel}(0)t + \frac{1}{2}\left(\frac{qE_{\parallel}}{m}\right)t^2$$

- So, there is a uniform acceleration $\left(\frac{qE_{\parallel}}{m}\right)$ along B.

∴ then: Perpendicular:

angular acceleration $\leftarrow m\frac{d\vec{V}_{\perp}}{dt} = q(\vec{E}_{\perp} + \vec{V}_{\perp} \times \vec{B})$

$$\vec{V}_{\perp} = \underbrace{\vec{\Omega}_c \times \vec{r}_c}_{} + \frac{\vec{E}_{\perp} \times \vec{B}}{B^2}$$

$$\frac{d^2 v_x}{dt^2} + \Omega_c^2 v_x = \Omega_c^2 \frac{E_y}{B}$$

$$v_x = -v_{\perp} \sin(\Omega_c t + \theta_0) + \frac{E_y}{B}$$

$$v_y = v_{\perp} \cos(\Omega_c t + \theta_0) - \frac{E_x}{B}$$

- So, a circular motion in the plane normal to B with constant velocity Ω_c and ~~constant~~ uniform motion perpendicular to E & B with $V_E\left(\frac{\vec{E}_{\perp} \times \vec{B}}{B^2}\right)$.

– the resultant motion:

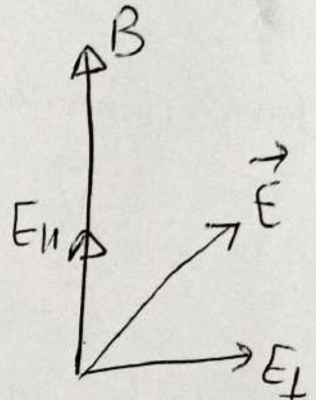
$$\vec{V}(t) = \underbrace{\vec{\Omega}_c \times \vec{r}_c}_{\text{rotation}} + \overbrace{\frac{\vec{E}_\perp \times \vec{B}}{B^2}}^{\text{constant drift}} + \overbrace{\left(\frac{q\vec{E}_{\parallel}}{m}\right)t}^{\text{linear acceleration}} + \overbrace{\vec{V}_{\parallel}(0)}^{\text{translation}}$$

angular acceleration

$$\dot{x}(t) = -\frac{\dot{V}_\perp}{\Omega_c}\cos(\Omega_c t + \theta_0) + \frac{E_y}{B}t + X_0 \quad ; \quad \vec{V}_\perp = \Omega_c \times r_c$$

$$\dot{y}(t) = \frac{-\dot{V}_\perp}{\Omega_c}\sin(\Omega_c t + \theta_0) - \frac{E_x}{B}t + Y_0$$

$$\dot{Z}(t) = Z_0 + V_{\parallel}(0)t + \frac{1}{2}\left(\frac{qE_{\parallel}}{m}\right)t^2$$

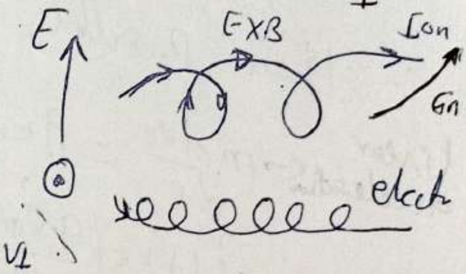


*** Note that:**

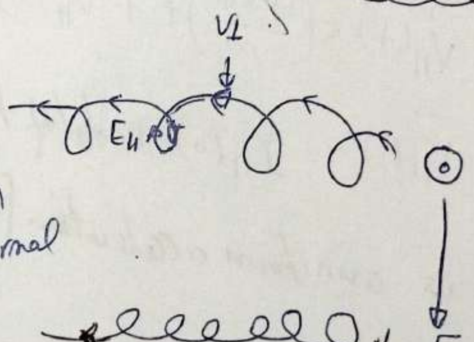
– $\Omega_c = \frac{|q|B}{m}$: depends on B only.

So, the angular velocity depends on B.

– $r_c = \frac{mV_\perp}{qB}$: depends on $V_\perp \propto E_\perp$ and B.



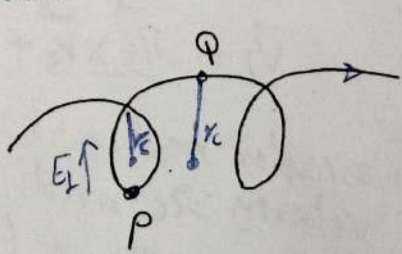
* trajectory:



(i) point P: $E_\perp$ and $(u_\perp \times B)$ acts in the upward direction, then the normal acceleration makes the trajectory more sharply bent than it would have been in the absence of $E_\perp$.

"Cycloidal trajectory"

(ii) point Q: $E_\perp$ in opposite of $u_\perp \times B$, so the normal acceleration decreases, so it becomes less bent.



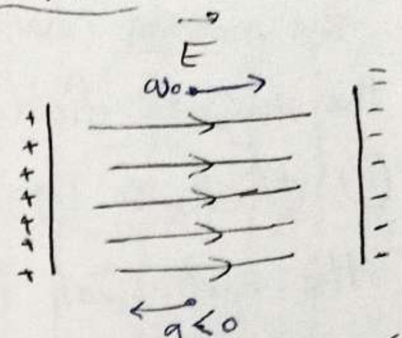
Also, at Q: $E_\perp$ increases $V_\perp$ and hence $r_c = \frac{mV_\perp}{|q|B}$ increases.

Summary:

uniform electrostatic field (constant ~~magnetic~~ Electric field):

① static charge:

- Positive charge will accelerate in the same direction of $\vec{E}$.
- Negative charge will accelerate in the opposite direction of $\vec{E}$



" Accelerated linear motion "

(ii) Moving charge:

- Same as static charge. " Accelerated "
- Same " " " translation (drift)

* Equations:

$$m\frac{d\vec{v}}{dt} = q\vec{E}$$

$$\vec{v}(t) = \vec{v}_0(0) + \left(\frac{q\vec{E}}{m}\right)t \qquad a = \frac{q\vec{E}}{m}.$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\left(\frac{q\vec{E}}{m}\right)t^2.$$

② Uniform magnetostatic field (constant magnetic field)

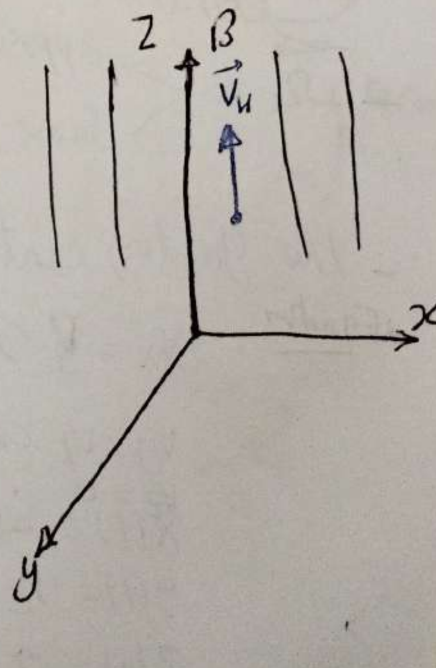
① static charge:

- whether positive or negative charge will not affected.

(ii) moving in same direction of B

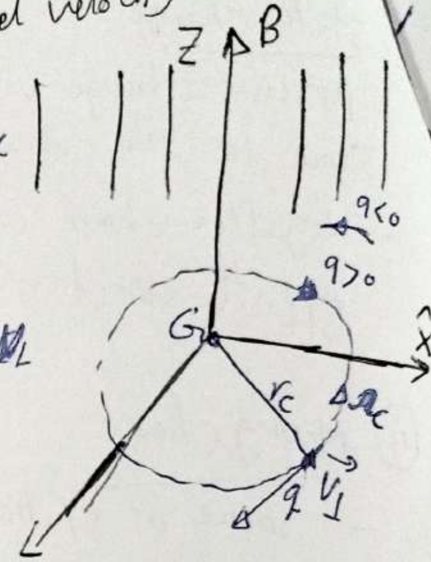
- the charge will move along the magnetic field with same initial velocity.

$$\vec{V}_{\parallel}(t) = \vec{V}_{\parallel}(0).$$



(iii) moving charge in the plane perpendicular normal to magnetic field.

- the moving charge will rotate around the magnetic field line → No initial Parallel velocity
  (i) clockwise ($q>0$) (ii) Counterclockwise ($q<0$).



- the radius of the circle

$$r_c = \frac{mV_\perp}{|q|B} = \frac{V_\perp}{\Omega_c}$$

$E_\perp$ can chase $V_\perp$

- is same as long as $V_\perp$ & B are constant and uniform. ($B(t)$, $B(x)$)

- the angular velocity around $\vec{B}$ is Ω_c

$$\Omega_c = \frac{|q|B}{m}$$

- is same as long as $\vec{B}$ is constant and uniform ($B(t)$, $B(x)$)

- there is no drift along the magnetic field since $\vec{V}_\parallel(t) = \vec{V}_\parallel(0) = 0$.

- uniform circular motion only < same radius r_c / same angular velocity Ω_c.
  gyration.

- $\vec{\Omega}_c$ is < opposite to B ⇒ $q>0$: clockwise rotation. / same to B → $q<0$: counterclockwise rotation.

- the guiding center position doesn't change.

*Equation:

$V_x = V_\perp \sin(\Omega_c t + \theta_0)$; $V_\perp^2 = V_x^2 + V_y^2$; $\vec{V}_\perp = \vec{\Omega}_c \times \vec{r}_c$.

$V_y = V_\perp \cos(\Omega_c t + \theta_0)$

$V_z = 0$

$X(t) = -r_c \cos(\Omega_c t) + X_0$; $r_c = \frac{V_\perp}{\Omega_c}$

$y(t) = r_c \sin(\Omega_c t) + Y_0$ | $(X-X_0)^2 + (Y-Y_0)^2 = r_c^2$.

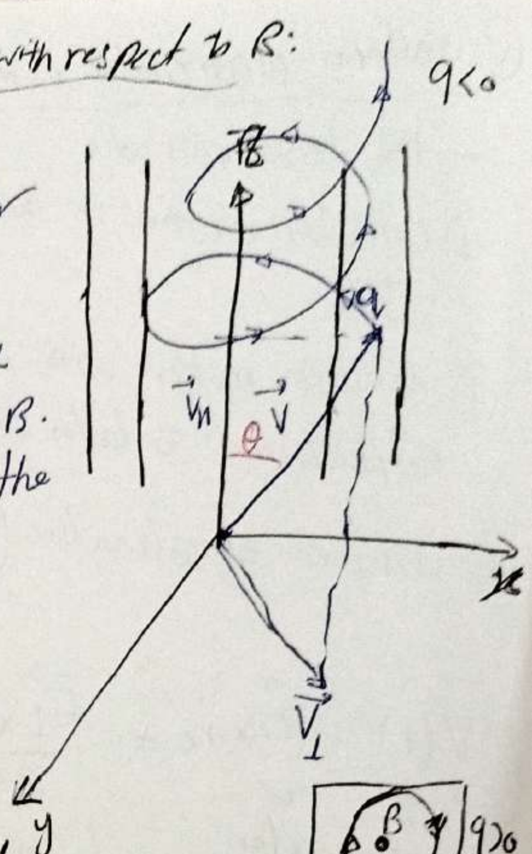
$Z(t) = Z_0$

moving charge in arbitrary oblique direction with respect to B:

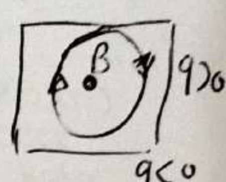
If there is initial velocity $\vec{V}(\theta)$,

So, there will be parallel and perpendicular component velocity:

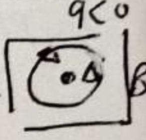
- the perpendicular component will make the circular motion in a plane normal to B.
- the parallel component will make the linear motion along B.



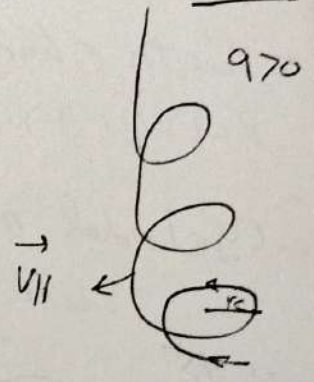
- Helical motion = circular motion (gyration) + linear motion



- cylindrical helix: if r_c is constant.
  Ω_c " " .
  $V_\parallel$ " " .



- the guiding center will drift along the magnetic field.

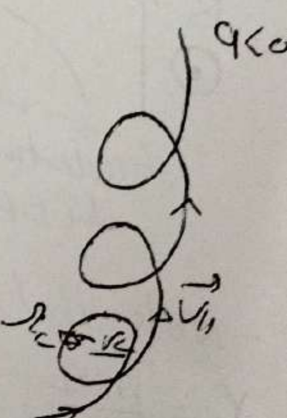


* Equations:

$$\vec{V}_x = \vec{V}_\perp \sin(\Omega_c t + \theta_0)$$

$$\vec{V}_y = V_\perp \cos(\Omega_c t + \theta_0)$$

$$\vec{V}_z = \vec{V}_z(0) = V_\parallel$$



$$x(t) = -r_c \cos(\Omega_c t + \theta_0) + X_0$$

$$y(t) = r_c \sin(\Omega_c t + \theta_0) + Y_0$$

$$Z(t) = V_\parallel(0) t + Z_0$$

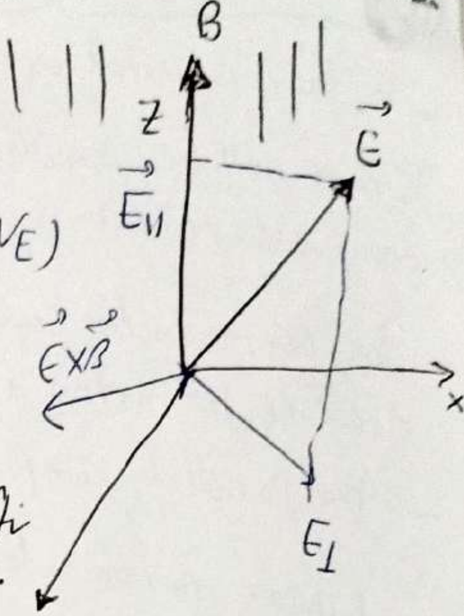
(V) uniform electrostatic and magnetic fields:

- the charge will do

(1) circular motion in the plane normal to B.

(2) uniform motion with constant velocity (V_E) perpendicular to both E and B.

(3) uniform acceleration $\left(\frac{qE_{\parallel}}{B}\right)$ along B.



$$\vec{V}(t) = \vec{\Omega}_c \times \vec{r}_c + \frac{\vec{E}_{\perp}\times\vec{B}}{B^2} + \left(\frac{q\vec{E}_{\parallel}}{m}\right)t + V_{\parallel}(0)$$

uniform acceleration along B.

circular motion in plane normal to B

uniform motion perpendicular to E & B

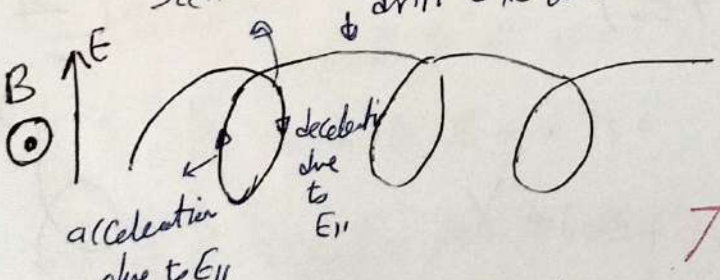
initial velocity along $\vec{B}$.

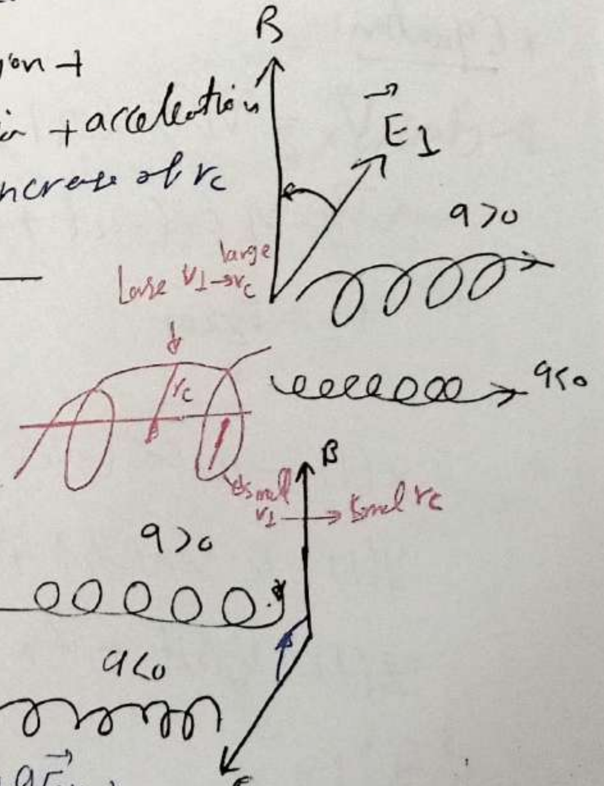
- Both charge will drift in the same direction V_E: doesn't depend on q & m.

But (1) $q>0$: clockwise (1) $q<0$: counter clockwise

- Cycloidal motion = circular motion + linear motion + acceleration

$\Omega_c \times r_c$. drift $E\times B$ due to increase of r_c





- Note that ($E\times B$) drift due to $r_c = \frac{mV_{\perp}}{|q|B}$, so the $V_{\perp}$ increases due to $E_{\perp}$: $m\frac{d\vec{V}_{\perp}}{dt} = q(E_{\perp} + V_{\perp}\times B)$

$$\vec{V}_{\perp} = \vec{\Omega}_c \times \vec{r}_c + \frac{\vec{E}\times\vec{B}}{B^2}; \quad \vec{V}_{\parallel}(t) = \left(\frac{q\vec{E}_{\parallel}}{m}\right)t + V_{\parallel}(0)$$