Nonlinear ion-acoustic waves in magnetized plasma at Venus' ionosphere

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Outlines

- **Introduction**
- **Ion-Acoustic Waves in Magnetized Cold Plasma at Venus' Ionosphere**
- **Nonlinear ion-acoustic waves in magnetized Warm plasma at Venus' ionosphere**

Why Waves?

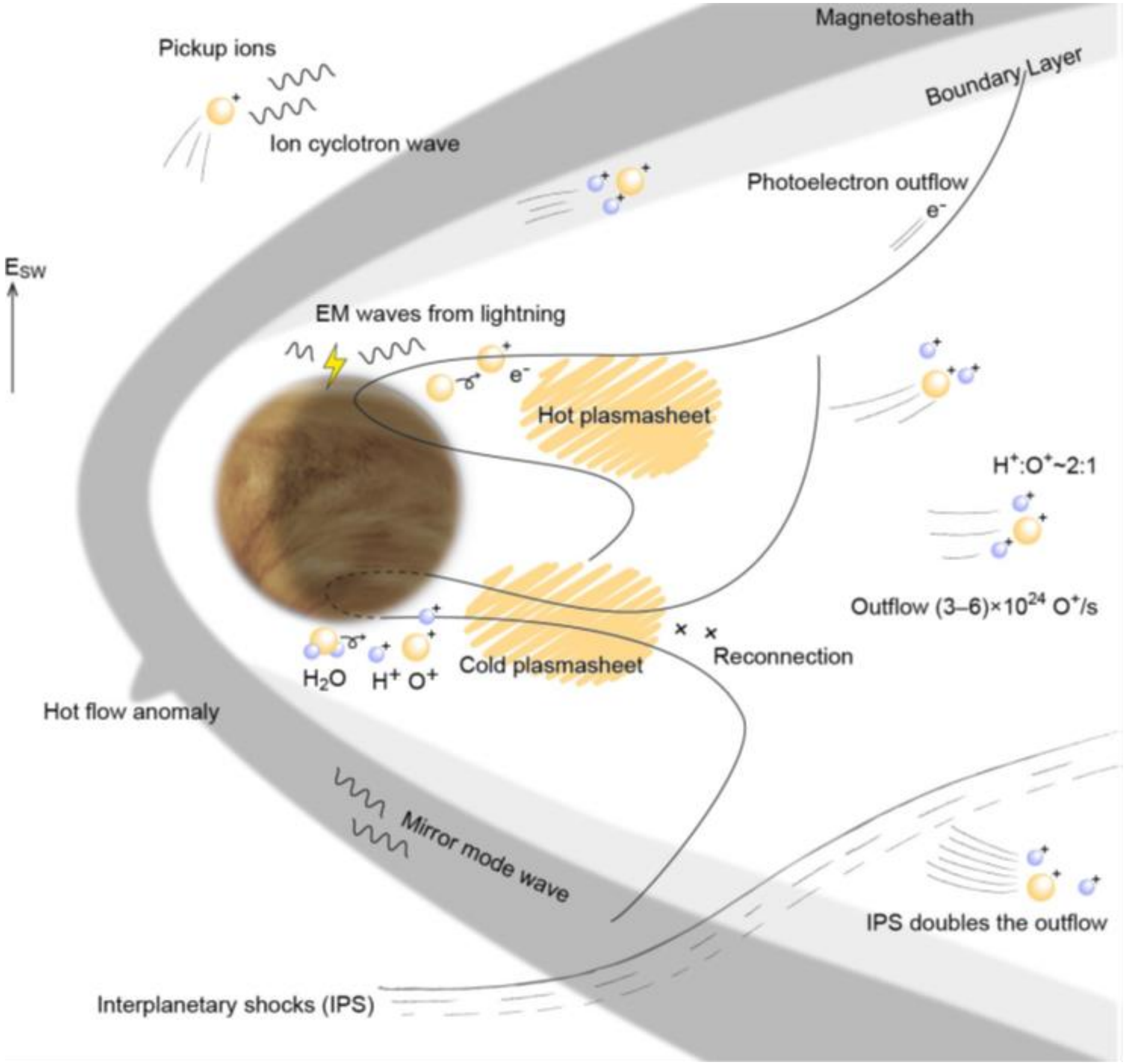
What is the importance to study waves in plasma?

(a) Plasma fingerprints appear in wave emissions . Thus, they are useful in faraway or unavailable plasma observation. They can serve as diagnostic tools.

(b) Plasma waves are essential for several processes , including energy transfer, ionospheric loss, particle acceleration, lightning, and heating

Planet Venus

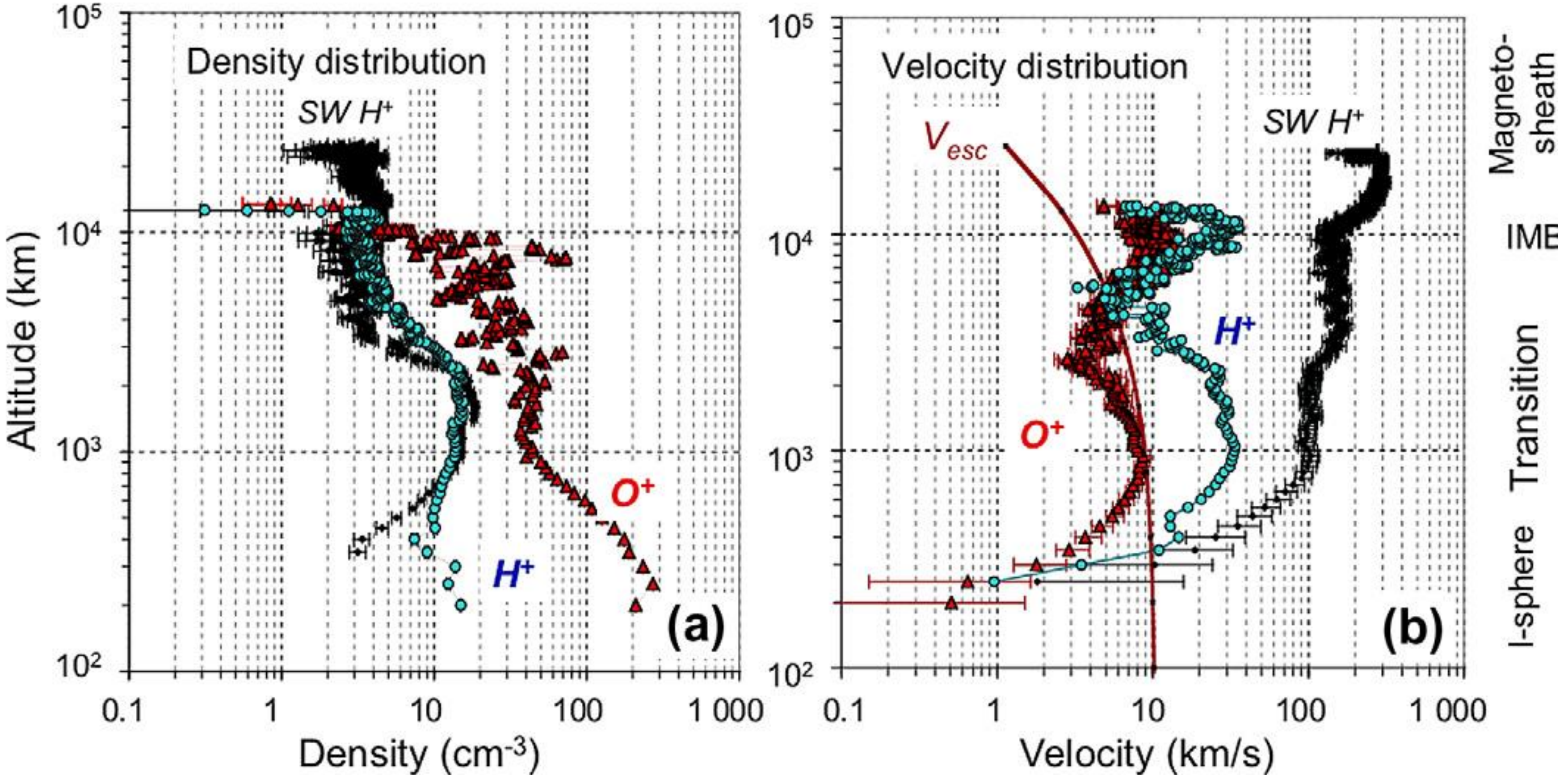
- Venus lacks an intrinsic magnetic field.
- Direct interaction between the solar wind and Venus.
- The solar radiation interacts directly with Venus' dense atmosphere forming a partially ionized shell called the *ionosphere*.
- Venus has a rich environment that can support the generation of different plasma modes.
- One of these basic modes is the ion-acoustic wave (IAW), which is the motivation of this work.



Missions of Venus

- **VEX** **(Venus express)**
- **PVO** **(pioneer Venus orbit)**

The density and velocity altitude profiles measured in the Dawn-Dusk meridian at the Venusian ionosphere by VEX



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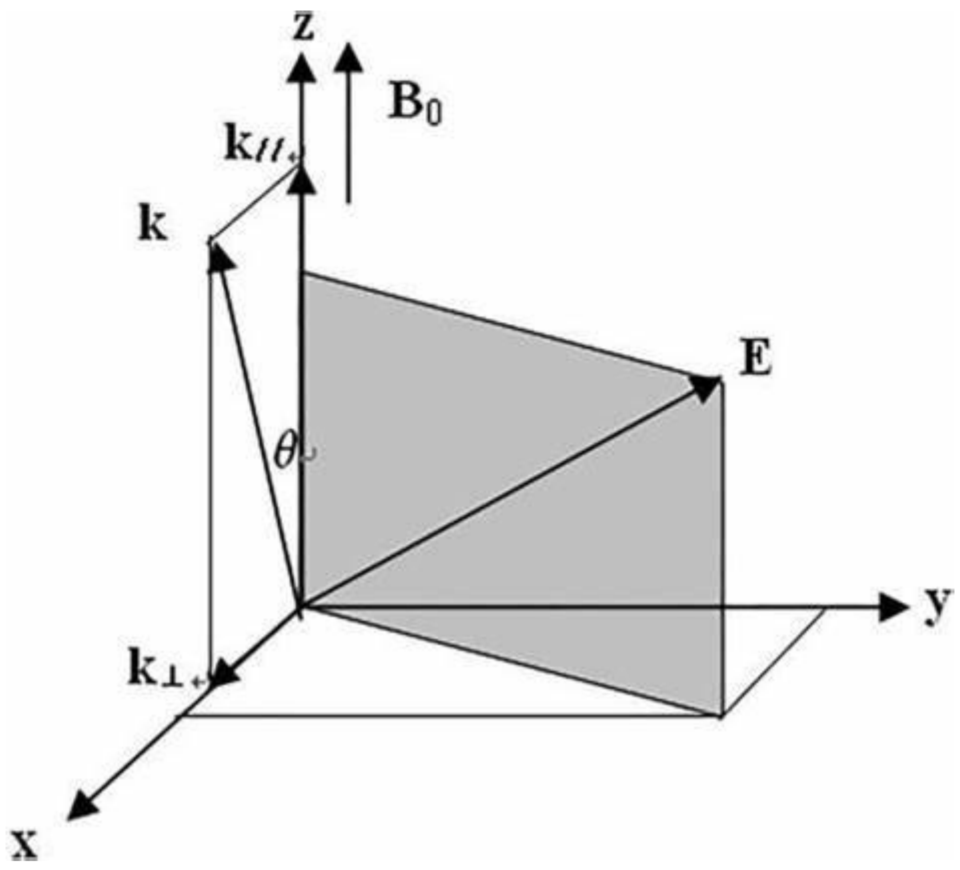
Ion-Acoustic Waves in Magnetized Cold Plasma at Venus' Ionosphere

Aim of the work

investigate the problem of small-amplitude ion acoustic waves in a cold plasma with two positive ions and electron with Maxwellian distribution using a reductive perturbation method (the ZK equation is derived and solved using the direct method)



Model Equations

Continuity equation

$$\partial_t n_\alpha + \nabla.(n_\alpha u_\alpha) = o$$

Equation of motion

$$m_\alpha n_\alpha(\partial_t u_a + u_\alpha \nabla. u_\alpha) - q n_{\boldsymbol{\alpha}}(\text{-}\nabla\varphi + {}^1/_{c_0}\, u_\alpha \times B) = o$$

Electric force   Magnetic force

Boltzmann distribution

$$n_e = n_{e_0} \exp\left(\frac{\mathbf{e}\phi}{k_B T\mathbf{e}}\right)$$

The system of equations is closed by Poisson equation

Where:

$\alpha = H^+$ and O^+

$$\nabla^2\phi = 4\pi \mathrm{e}(n_e - n_0 - n_H)$$

The normalized system of equations

The nonlinear dynamics of IAWs are governed by the following set of normalized hydrodynamic equations

$$\partial_t n_O + \partial_x (n_O u_{Ox}) + \partial_y (n_O u_{Oy}) + \partial_z (n_O u_{Oz}) = 0$$

$$\partial_t u_{Ox} + u_{Ox}\partial_x u_{Ox} + u_{Oy}\partial_y u_{Ox} + u_{Oz}\partial_z u_{Ox} + \partial_x \phi - w_{sO} u_{Oy} = 0$$

$$\partial_t u_{Oy} + u_{Ox}\partial_x u_{Oy} + u_{Oy}\partial_y u_{Oy} + u_{Oz}\partial_z u_{Oy} + \partial_y \phi + w_{sO} u_{Ox} = 0$$

$$\partial_t u_{Oz} + u_{Ox}\partial_x u_{Oz} + u_{Oy}\partial_y u_{Oz} + u_{Oz}\partial_z u_{Oz} + \partial_z \phi = 0$$

$$\partial_t n_H + \partial_x (n_H u_{Hx}) + \partial_y (n_H u_{Hy}) + \partial_z (n_H u_{Hz}) = 0,$$

$$\partial_t u_{Hx} + u_{Hx}\partial_x u_{Hx} + u_{Hy}\partial_y u_{Hx} + u_{Hz}\partial_z u_{Hx} + \mu_H \partial_x \phi - \mu_H w_{sH} u_{Hy} = 0$$

$$\partial_t u_{Hy} + u_{Hx}\partial_x u_{Hy} + u_{Hy}\partial_y u_{Hy} + u_{Hz}\partial_z u_{Hy} + \mu_H \partial_y \phi + \mu_H w_{sH} u_{Hx} = 0$$

$$\partial_t u_{Hz} + u_{Hx}\partial_x u_{Hz} + u_{Hy}\partial_y u_{Hz} + u_{Hz}\partial_z u_{Hz} + \mu_H \partial_z \phi = 0$$

The normalized system (cont.)

The electrons are described by Maxwellian distribution as

$$n_e = \exp(\phi)$$

The system is closed by Poisson equation

$$(\partial^2 x + \partial^2 y + \partial^2 z)\phi - \delta_e n_e + n_0 + \delta_H n_H = 0$$

$$\mu_H = \frac{m_0}{m_H} \qquad \delta_e = \frac{n_e^{(0)}}{n(0)}, \delta_H = \frac{n_H^{(0)}}{n(0)} \qquad \omega_{si} = \frac{\omega_{c_i}}{\omega p_0}$$

the relative ion mass

the relative densities

the relative frequency

Derivation of Zakharov_ Kuznetsov

To investigate the proliferation of IAWs, the dependent variables in equations (ref1)–(ref10) are enlarged about their equilibrium values in terms of epsilon as

$$n_j = 1 + \epsilon n_j^{(1)} + \epsilon^2 n_j^{(2)} + \epsilon^3 n_j^{(3)} + ...,$$

$$u_{ix} = \epsilon^{3/2} u_{ix}^{(1)} + \epsilon^2 u_{ix}^{(2)} + \epsilon^{5/2} u_{ix}^{(3)} + ...,$$

$$u_{iy} = \epsilon^{3/2} u_{iy}^{(1)} + \epsilon^2 u_{iy}^{(2)} + \epsilon^{5/2} u_{iy}^{(3)} + ...,$$

$$u_{iz} = \epsilon u_{iz}^{(1)} + \epsilon^2 u_{iz}^{(2)} + \epsilon^3 u_{iz}^{(3)} + ...,$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + ...,$$

- the independent variables can be expressed as in a moving frame in which the nonlinear structure propagates with a phase speed of V

$$\xi = \epsilon^{(1/2)}(z - Vt), \qquad \eta = \epsilon^{1/2} y, \qquad \zeta = \epsilon^{1/2} x, \qquad \text{and} \qquad \tau = \epsilon^{3/2} t,$$

$$n_O^{(1)} = \frac{1}{V^2}\phi^{(1)}, \qquad n_H^{(1)} = \frac{-\mu_H}{V^2 \mu_H}\phi^{(1)},$$

$$u_{Ox}^{(1)} = \frac{-1}{w_{sO}}\partial_\eta \phi^{(1)}, \qquad u_{Hx}^{(1)} = \frac{-\mu_H}{w_{sH}}\partial_\eta \phi^{(1)},$$

$$u_{Oy}^{(1)} = \frac{1}{w_{sO}}\partial_\zeta \phi^{(1)}, \qquad u_{Hy}^{(1)} = \frac{\mu_H}{w_{sH}}\partial_\zeta \phi^{(1)},$$

$$u_{Oz}^{(1)} = \frac{1}{V}\phi^{(1)}, \qquad u_{Hz}^{(1)} = \frac{-\mu_H}{V}\phi^{(1)},$$

$$n_e = \phi^{(1)}.$$

Derivation of Zakharov_ Kuznetsov

we obtain an evolution nonlinear partial differential equation in the form of the ZK equation for the first-order perturbed electrostatic potential as

$$\partial_\tau \phi^{(1)} + A\phi^{(1)}\partial_\xi \phi^{(1)} + B\partial_\xi^3 \phi^{(1)} + C\partial_\xi \left(\partial_\zeta^2 + \partial_\eta^2\right)\phi^{(1)} = 0$$

where

$$A = B\left(\frac{3}{v^4} + \frac{3\delta_H \mu_H^2}{v^4} - \delta_e\right)$$

$$B = \left(\frac{2}{v^3} + \frac{2\delta_H \mu_H}{v3}\right)^{-1}$$

$$C = B_1 + \frac{1}{\omega_{s_0}^2} + \frac{\delta H \mu_H}{\omega_{SH}^2}$$

Analytical solutions of Zakharov_ Kuznetsov

Using the direct soliton method According to Z-k equation will be transformed to an ordinary differential equation (ODE) using the travelling transformation $Y = L_1\xi + L_2\eta + L_3\zeta - M\tau$, where M represents the frame velocity, and the direction cosines $L_1^2 + L_2^2 + L_3^2 = 1$. Thus, Z-K equation is written as

$$\frac{d^3\rho}{dY^3} + H_1\rho\frac{d\rho}{dY} + H_2\frac{d\rho}{dY} = 0$$

ρ can be expressed as $\rho(\xi, \eta, \zeta, \tau) = \rho(Y)$

$$\rho(Y) = \frac{1}{H_3}\operatorname{sech}^2\left(\frac{H_2}{4}\right)^{\frac{1}{2}} Y$$

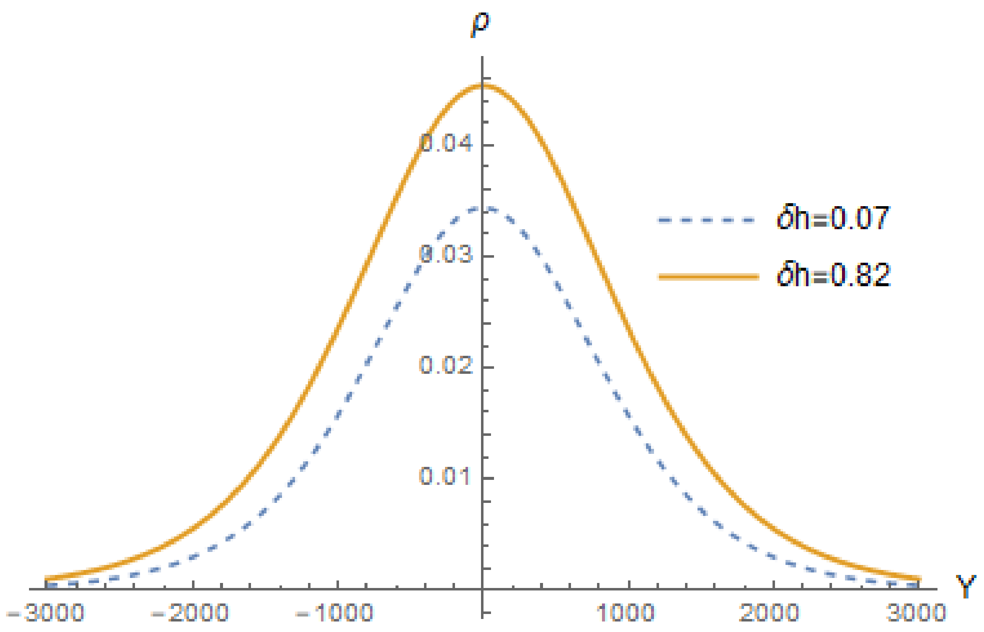
where H_3 are constants determined from $2H_1/\ 3H_2$

the soliton amplitude $1/H_3$

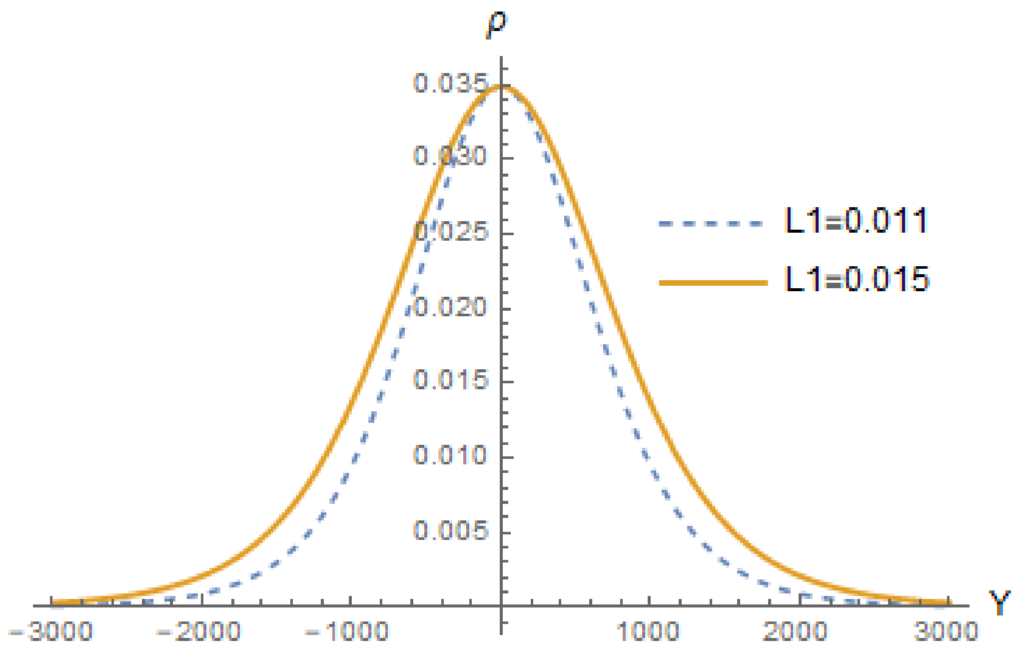
and the soliton width $\left(\frac{H_2}{4}\right)^{\frac{1}{2}} Y$

the transformed coordinate with respect to a frame moving with velocity V is Y (i.e.$Y = \zeta - V\tau$)

Soliton wave



The soliton profile is illustrated for different values of the relative density δ_H,where plasma parameters are: $B_0 = 85 \times 10^{-5}$ nT, $L_1 = L_2 = 0.011$, and
n_o = [250 – 73] cm^{-3}, n_H =[17 – 60]cm^{-3}



The soliton profile is illustrated for different values of direction cosineL_1,whereplasma parameters are: $B_0 = 85 \times 10^{-5}$ nT, $L_2 = 0.01$

Summery

- The plasma model consists of two positive H^{+}and0^{+} separately charged cold ions and Maxwellian electrons.
- The direct reductive method to achieve a set of moving wave solutions characterizing nonlinear waves soliton.
- Analytical solutions of the wave amplitude exposes perturbation potential, that is, wave solitons are composed in the basis of Venus ionosphere at region $(200-1000)$ km.
- The influence of plasma parameters such as density ratio δ_H and the direction cosine L_1 on the qualities of soliton format has been considered.

Nonlinear ion-acoustic waves in magnetized Warm plasma at Venus' ionosphere

Aim of the work

In Venusian magneto sheath , Venus express has observed the existence of large amplitude nonlinear waves which being small waves developing by oscillations to unstable region then large amplitude.

The novelty of the present work is to find the proper conditions that allow different nonlinear waves such as soliton, blow up, and shock-like waves to exist in the Venusian ionosphere.

Basic Equations

Continuity equation

$$\partial_t n_\alpha + \nabla . (n_\alpha u_\alpha) = o$$

Equation of motion

$$m_\alpha n_\alpha(\partial_t u_a + u_\alpha \nabla u_\alpha) - q n_{\boldsymbol{\alpha}}(-\nabla\varphi + {}^1/_{c_0}\, u_\alpha \times B) + \nabla P_\alpha = o$$

Electric force Magnetic force Pressure gradient force

Boltzmann distribution

$$n_e = n_{e_0} \exp\left(\frac{e\phi}{k_B Te}\right)$$

The system of equations is closed by Poisson equation

$$\nabla^2\phi = 4\pi e(n_e - n_0 - n_H)$$

Where

$\alpha = H^+ \text{and} 0^+$

$$\nabla P_\alpha = k_B\, T_\alpha \nabla\, n_\alpha$$

Derivation of the evolution equations

$$\partial_t n_O + \partial_x(n_O u_{Ox}) + \partial_y(n_O u_{Oy}) + \partial_z(n_O u_{Oz}) = 0, \qquad (1)$$

$$\partial_t u_{Ox} + u_{Ox}\partial_x u_{Ox} + u_{Oy}\partial_y u_{Ox} + u_{Oz}\partial_z u_{Ox} + \partial_x\phi - w_{sO}u_{Oy} + \frac{5}{3}\sigma_O n_O^{-1/3}\partial_x n_O = 0, \qquad (2)$$

$$\partial_t u_{Oy} + u_{Ox}\partial_x u_{Oy} + u_{Oy}\partial_y u_{Oy} + u_{Oz}\partial_z u_{Oy} + \partial_y\phi + w_{sO}u_{Ox} + \frac{5}{3}\sigma_O n_O^{-1/3}\partial_y n_O = 0, \qquad (3)$$

$$\partial_t u_{Oz} + u_{Ox}\partial_x u_{Oz} + u_{Oy}\partial_y u_{Oz} + u_{Oz}\partial_z u_{Oz} + \partial_z\phi + \frac{5}{3}\sigma_O n_O^{-1/3}\partial_z n_O = 0, \qquad (4)$$

$$\partial_t n_H + \partial_x(n_H u_{Hx}) + \partial_y(n_H u_{Hy}) + \partial_z(n_H u_{Hz}) = 0, \qquad (5)$$

$$\partial_t u_{Hx} + u_{Hx}\partial_x u_{Hx} + u_{Hy}\partial_y u_{Hx} + u_{Hz}\partial_z u_{Hx} + \mu_H\partial_x\phi - \mu_H w_{sH}u_{Hy} + \frac{5}{3}\mu_H\sigma_H n_H^{-1/3}\partial_x n_H = 0, \qquad (6)$$

$$\partial_t u_{Hy} + u_{Hx}\partial_x u_{Hy} + u_{Hy}\partial_y u_{Hy} + u_{Hz}\partial_z u_{Hy} + \mu_H\partial_y\phi + \mu_H w_{sH}u_{Hx} + \frac{5}{3}\mu_H\sigma_H n_H^{-1/3}\partial_y n_H = 0, \qquad (7)$$

$$\partial_t u_{Hz} + u_{Hx}\partial_x u_{Hz} + u_{Hy}\partial_y u_{Hz} + u_{Hz}\partial_z u_{Hz} + \mu_H\partial_z\phi + \frac{5}{3}\mu_H\sigma_H n_H^{-1/3}\partial_z n_H = 0. \qquad (8)$$

Derivation of the evolution equations (cont.)

The electrons are described by Maxwellian distribution as

$$n_e = \exp(\phi). \tag{9}$$

Equations (1)–(9) are closed by Poisson equation

$$\left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\phi - \delta_e n_e + n_O + \delta_H n_H = 0. \tag{10}$$

where

$$\sigma_0 = \frac{T_0}{Te} \qquad \sigma_H = \frac{T_H}{Te}$$

We introduce the stretched space-time coordinates *X; Y; Z*, and *T* as

$X = \varepsilon^{1/2}(x - Vt), \quad Y = \varepsilon^{\frac{1}{2}} y, \quad Z = \varepsilon^{\frac{1}{2}} z, \quad \text{and } T = \varepsilon^{2/3} t$

where V is the IAW phase velocity and $\boldsymbol{\varepsilon}$ is a small parameter that measures the size of the perturbation amplitude. Let us expand the variables *nj*, *uix*, *uiy*, *uiz*, and $\boldsymbol{\phi}$ in powers of ϵ

$$
\begin{aligned}
n_j &= 1 + \epsilon n_j^{(1)} + \epsilon^2 n_j^{(2)} + \epsilon^3 n_j^{(3)} + \ldots, \\
u_{ix} &= u_{ix}^{(0)} + \epsilon u_{ix}^{(1)} + \epsilon^2 u_{ix}^{(2)} + \epsilon^3 u_{ix}^{(3)} + \ldots, \\
u_{iy} &= \epsilon^{3/2} u_{iy}^{(1)} + \epsilon^2 u_{iy}^{(2)} + \epsilon^{5/2} u_{iy}^{(3)} + \epsilon^3 u_{iy}^{(4)} + \ldots, \\
u_{iz} &= \epsilon^{3/2} u_{iz}^{(1)} + \epsilon^2 u_{iz}^{(2)} + \epsilon^{5/2} u_{iz}^{(3)} + \epsilon^3 u_{iz}^{(4)} + \ldots, \\
\phi &= \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \ldots.
\end{aligned}
$$

Using the following basic set equations we obtain the ZK equation for the first-order perturbed potential

$$\frac{\partial\varphi}{\partial\tau}+A_1\varphi\frac{\partial\varphi}{\partial\zeta}+B\frac{\partial^3\varphi}{\partial\zeta^3}+A_2\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\varphi}{\partial\xi^2}+\frac{\partial^2\varphi}{\partial\eta^2}\right)=0.$$

where B is the dispersion is the form

$$B=\left(\frac{18V}{\left(3V^2-5\sigma_o\right)^2}+\frac{18V\delta_h\mu_h}{\left(3V^2-5\mu_h\sigma_h\right)^2}\right)^{-1},$$

and A_1, A_2 , are the nonlinear terms, in ZK equation are the forms

$$A_1=B\left(\frac{3\left(27V^2-5\sigma_o\right)}{\left(3V^2-5\sigma_o\right)^3}+\frac{3\delta_h\mu_h^2\left(27V^2-5\delta_h\mu_h\right)}{\left(3V^2-5\mu_h\sigma_h\right)^3}-\delta_e\right),$$

$$A_2=B\left(1+\frac{9V^4}{w_{so}^2\left(3V^2-5\sigma_o\right)^2}+\frac{9\delta_hV^4}{w_{sh}^2\mu_h\left(3V^2-5\mu_h\sigma_h\right)^2}\right),$$

we can use $\varphi_1(\xi,\eta,\zeta,\tau)=\varphi(\xi,\eta,\zeta,\tau)$.

Mathematical solutions

- we introduce a class of solutions for the nonlinear evolution equations is obtained by new Generalized (G'/G)-Expansion Method
- We apply this method to solve exact solutions for the ZK Eq. can be transform to ordinary differential Eq.

$$\frac{d^3\phi}{dX^3} + H_1\phi\frac{d\phi}{dX} - H_2\frac{d\phi}{dX} = 0,$$

where

$$H_1 = \frac{A_1 L_3}{BL_3^3 + A_2 L_3(L_1^2 + L_2^2)}, \qquad \text{and} \qquad H_2 = \frac{M}{BL_3^3 + A_2 L_3(L_1^2 + L_2^2)},$$

where $\phi^{(1)} = \phi$. Equation (21) can be Integrated with respect to X to obtain

$$\frac{d^2\phi}{dX^2} + \frac{H_1}{2}\phi^2 - H_2\phi = 0.$$

$$\phi(X) = \sum_{l=0}^{n} a_l \left(\frac{G'}{G}\right)^l + \sum_{l=1}^{n} b_l \left(F_l(X) + \frac{G'}{G}\right)^{-l}, \tag{24}$$

where $F_l(X)$ are functions in X, while a_l and b_l are constants. Using eq. (24) into (23) and following the usual procedure of the G'/G-expansion technique, we obtain The positive integer n can be determined by balancing the highest-order nonlinear terms with the highest-order derivatives in eq. (23). This gives us $n = 2$. Substituting eq. (24) into (23) and then collecting a power derivative of G, we get the constants in terms of the physical parameters

The function $G(X)$ satisfies the Riccati equation

$$\frac{d^2G}{dX^2} + \beta_1 \frac{dG}{dX} + \beta_2 G = 0, \tag{29}$$

where β_1 and β_2 are constants. Eq. (29) has a solution is

$$G(X) = \exp\left(\frac{-\beta_1}{2}X\right)(c_1\sinh[\theta_1 X] + c_2\cosh[\theta_1 X]), \quad (30)$$

with $c_1 > c_2$, $(\beta_1^2 - 4\beta_2^2)^{1/2} > 0$, and $\theta_1 = (\beta_1^2 - 4\beta_2^2)^{1/2}$,

$$G(X) = \exp\left(\frac{-\beta_1}{2}X\right)(c_3\sin[\theta_2 X] + c_4\cos[\theta_2 X]), \quad (31)$$

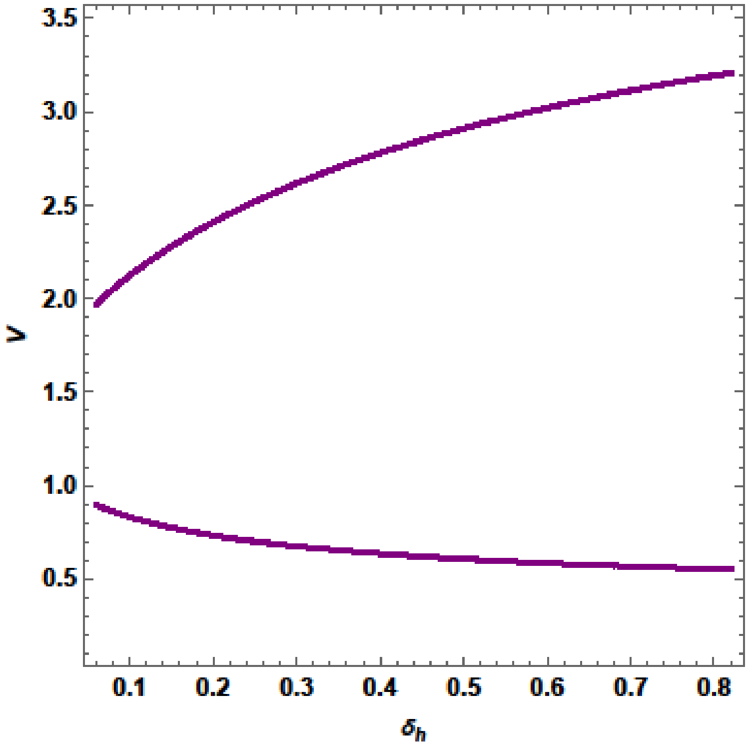
with $c_3 < c_4$, $(\beta_1^2 - 4\beta_2^2)^{1/2} < 0$, and $\theta_2 = (\beta_1^2 - 4\beta_2^2)^{1/2}/i$,

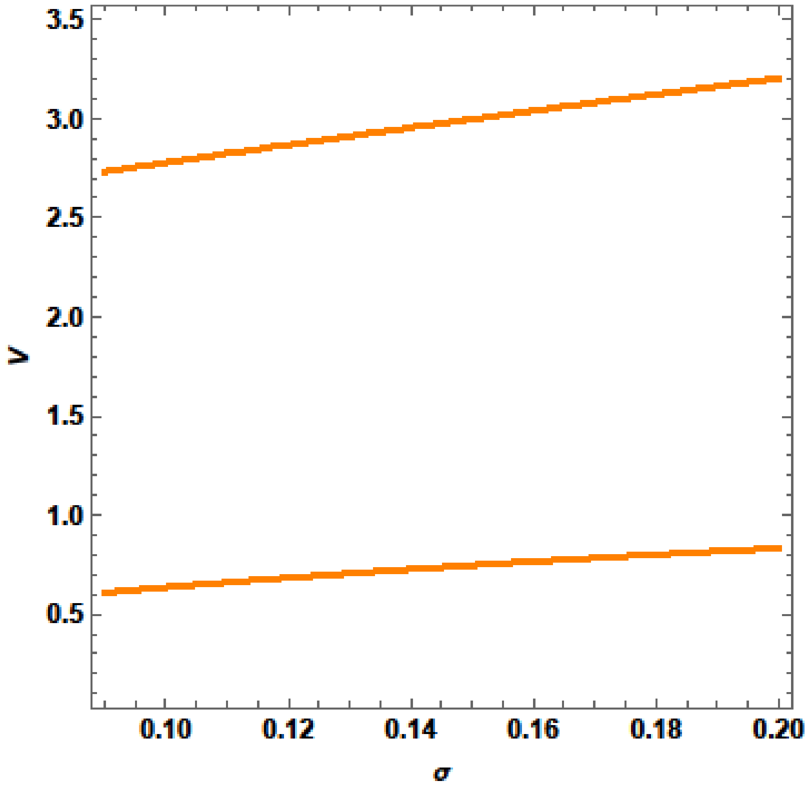
$$G(X) = \exp\left(\frac{-\beta_1}{2}X\right)(c_5 + Xc_6), \quad (32)$$

with $(\beta_1^2 - 4\beta_2^2)^{1/2} = 0$.

The phase velocity V against the relative density δ_H , where $\sigma_o = \sigma_H = 0{:}1$.

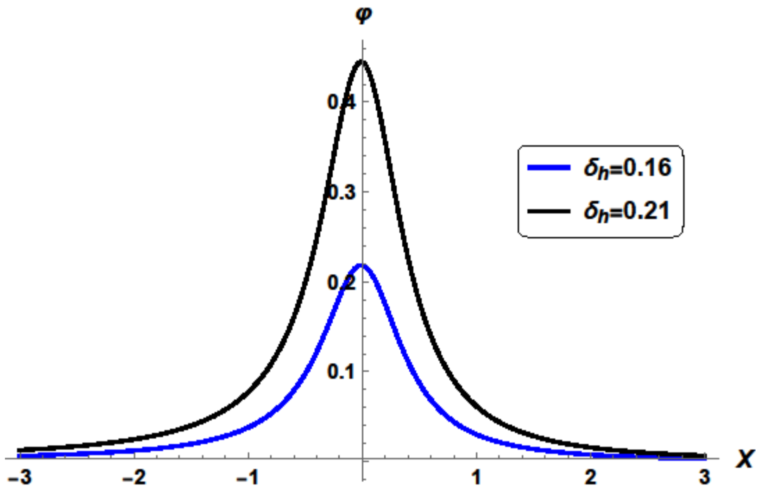
The phase velocity V against the relative temperature $\sigma(\sigma_o = \sigma_H)$, where $\delta_H = 0.4$

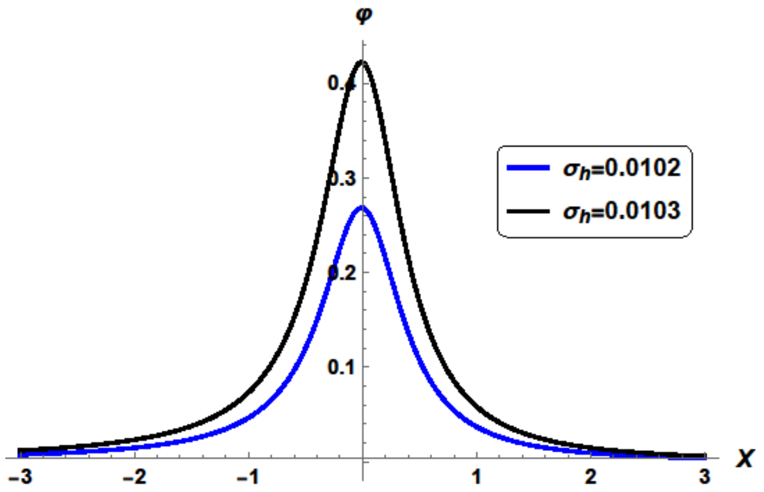


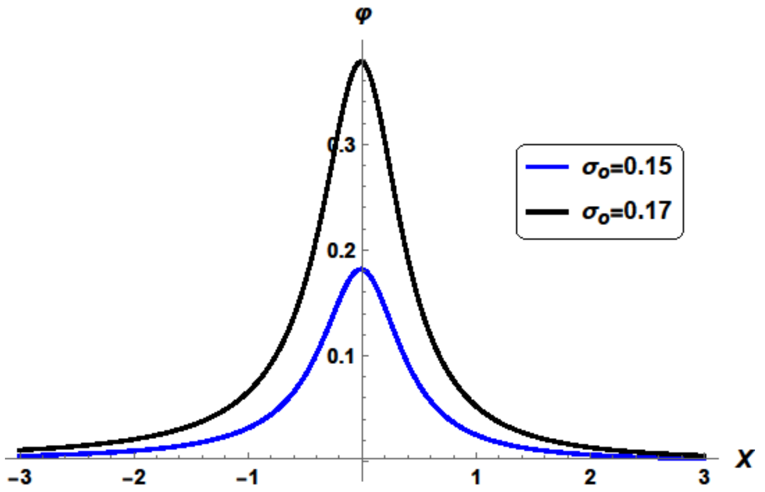


Soliton wave

- The soliton profile with different values of the relative density δ_H ,
  - the soliton profile with different values of the relative temperature σ_{H}
- the soliton profile with different values of the relative temperature σ_{o}

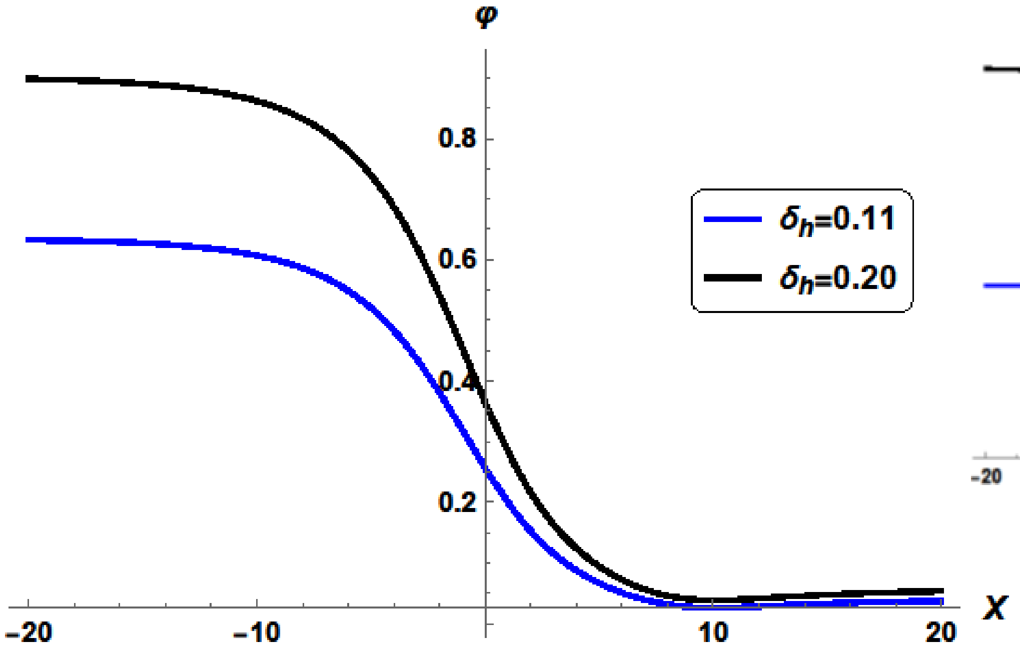


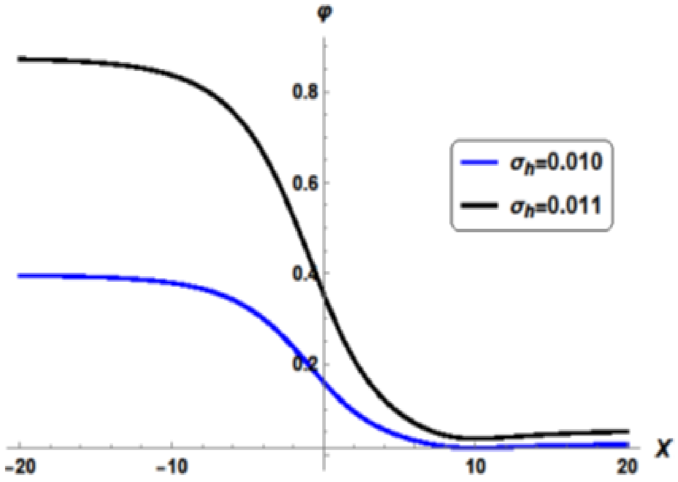


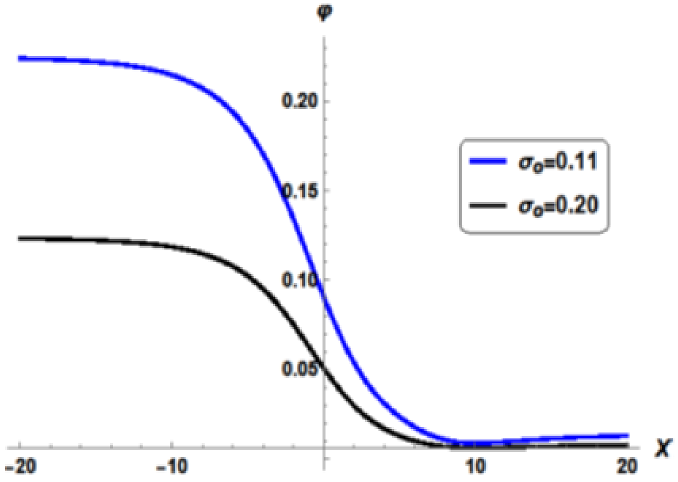


Shock-like wave

The profiles for the shock like wave against the normalized spatial coordinate, X, for different plasma parameters

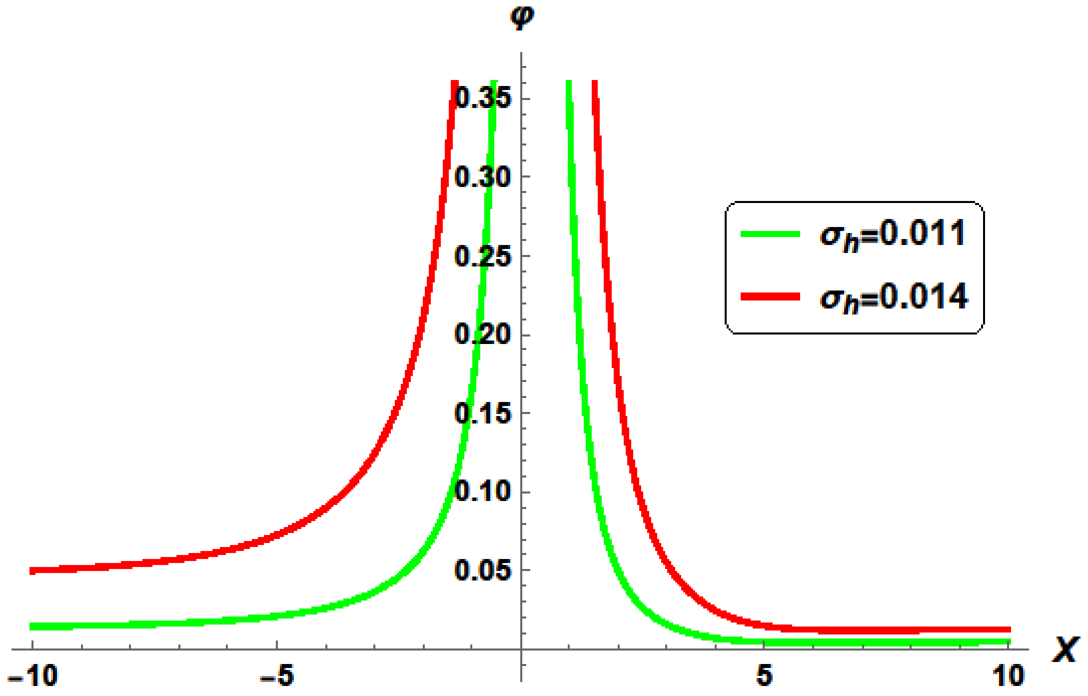


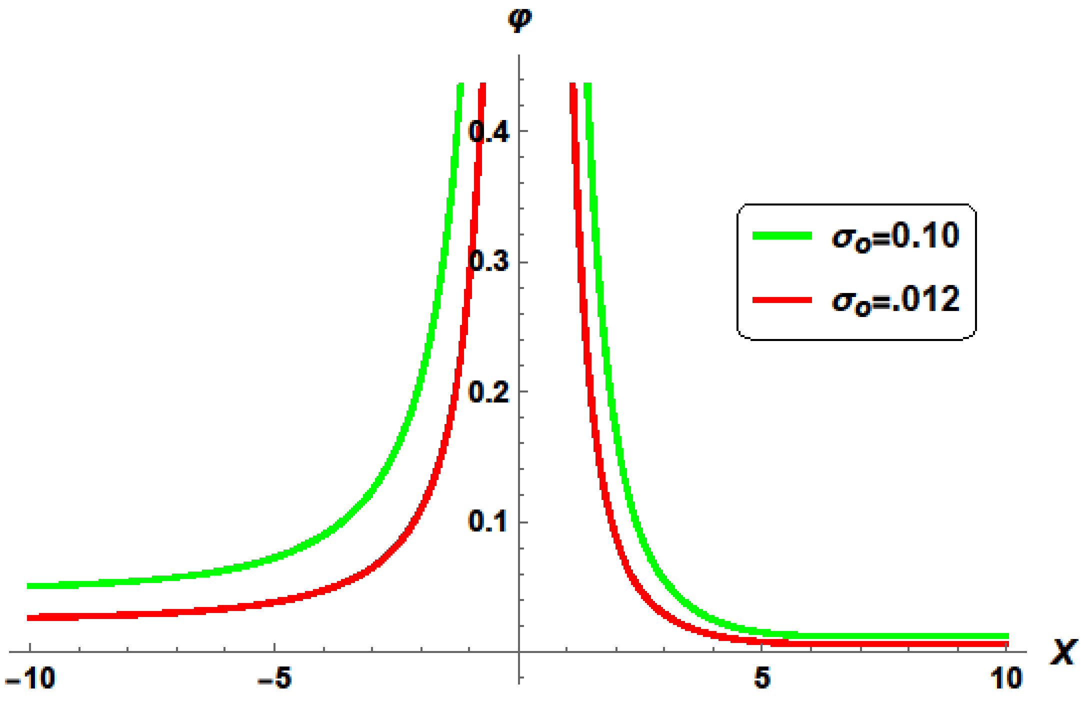




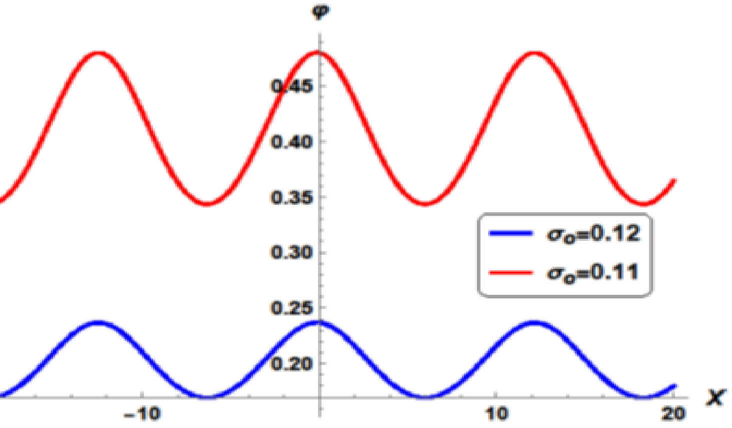
Blow up wave

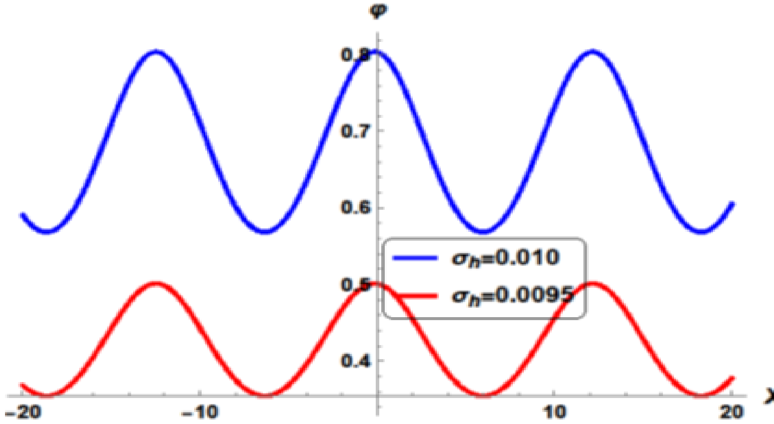
The profiles for the explosive pulse against the normalized spatial coordinate, X, for different values of the relative temperatures.

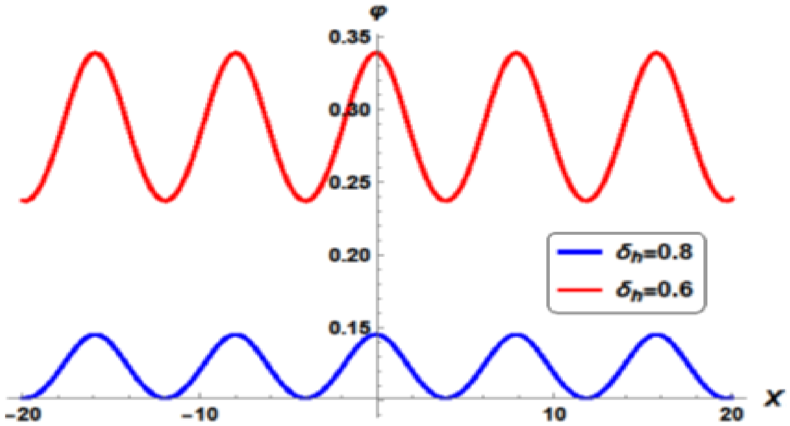




Periodical profile







- The periodical profile with different values of the relative density δ_H ,
- the periodical profile with different values of the relative temperature σ_{H}
- the periodical profile with different values of the relative temperature σ_{o}

Summery

- Soliton, shock like , periodic , and explosive waves, are explored in a collision-less, magnetized, multi-component plasma relevant to Venus' ionospheric (at altitudes 200–1000 km)
- The plasma model Consists of warm hydrogen $H^+$and oxygen 0^+ions as well as Maxwellian electrons.
- The nonlinear dynamics of these structures are described by the nonlinear Zakharov-Kuznetsov equation.
- The G`/G-expansion technique is employed to describe the nonlinear structures of interest.
- Subsonic and supersonic compressive ion-acoustic waves have been found.
- The results show that the ions concentration and temperature affect the basic features (phase velocity, amplitude, and width) of the pulses.

Published Research Papers

A. Elmandoh, A. A. Fayad , R. E. Tolba , and W. M. Moslem. [July 2023]. "Soliton, Blow up, and Shock-like Ion-Acoustic Waves in Magnetized Plasma at Venus' Ionosphere". In: Indian J Phys 97.8, pp. 2537–2545.