

# Earth's Ionosphere: Modeling and simulation

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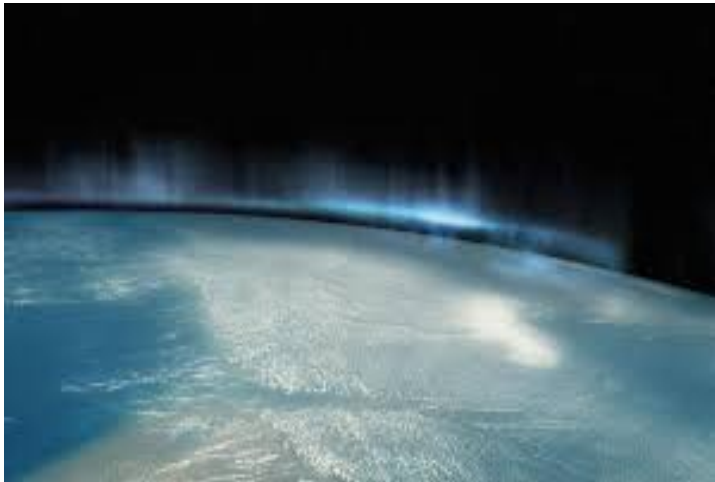
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Earth's ionosphere is an open laboratory that enables the in-situ study of plasma. Composed of myriad chemical species subject to ionization and recombination, it plays an important role in Earth's environment. The ionosphere's stratified layers protect Earth from harmful solar radiation and meteorites, and are crucial for communication using satellites and ground-based wave reflection. The Boltzmann equations are among the most important mathematical models for studying ionospheric processes, as the ionosphere contains many neutral and charged species. Ionospheric modeling combines fluid and transport equations, validated by data from satellite, rocket, and balloon instruments.

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# What is atmosphere?



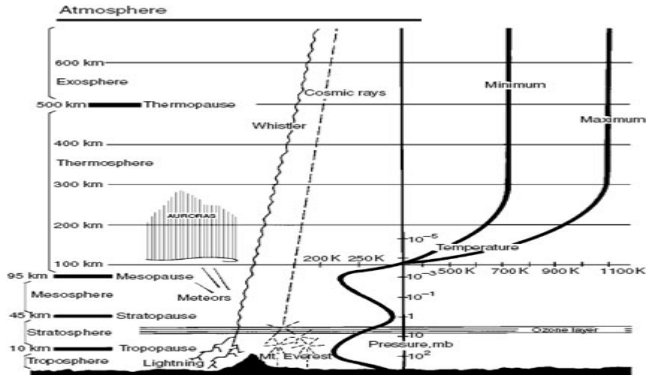
<https://www.youtube.com/watch?v=5C6hbf5rgjE>

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The ionosphere itself is also divided into three distinct regions that can change significantly between day and night depending on the intensity of ultraviolet (UV) radiation. These regions may even disappear entirely at night.



The ionosphere serves as a natural, open-air laboratory where we can directly study plasma. *in situ*.

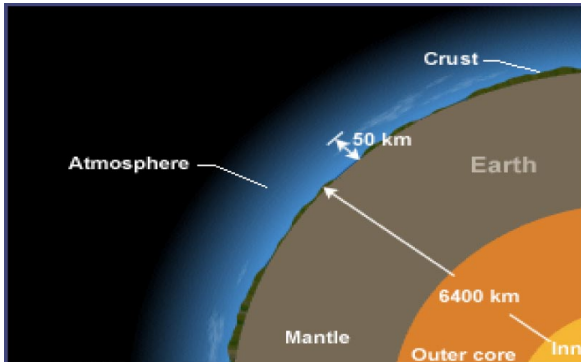
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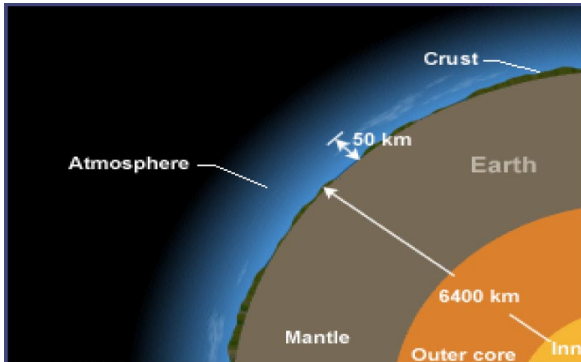
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The ionosphere, the ionized part of the Earth's upper atmosphere, consists of neutral atoms, free electrons, ions, and trace amounts of dust particles.

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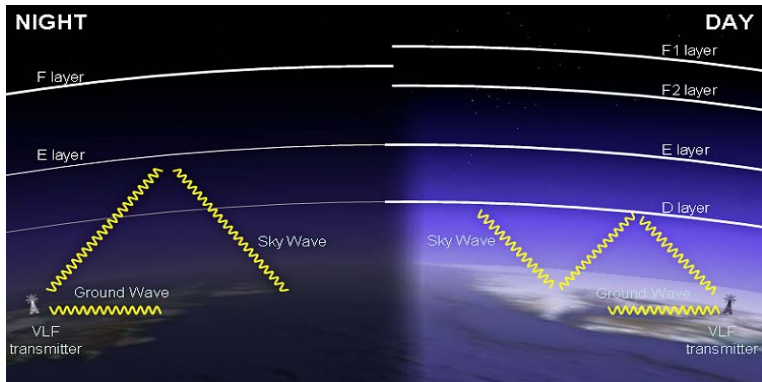
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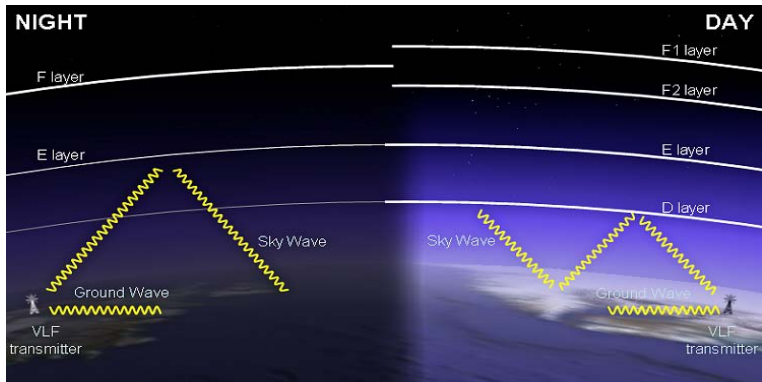
- Absorb part of (UV) radiations and heat the Earth surface.
- Reduce the temperature gradient between day and night
- Recycle the water.

- **D region:** ( 60-90 *km* altitude), electron density  $10^8 - 10^{10} m^{-3}$  and dominated by negative many-atomic ions. Exists only in the presence of sun rays. Absorb waves less than  $\sim MHz$  and reflects low frequency waves (30 to 300kHz).
- **E region:** ( 90-120 *km*), electron density  $\sim 10^{11} m^{-3}$ . Contains metallic ions,  $NO^+$  and  $O_2^+$ .
- **F region:** (150-500 *km*) the most ionized part of the ionosphere, electron density can reach  $10^{11}$  to  $10^{12} m^{-3}$  at altitude 250 *km*. The attitude change depending on the daytime. reflects high frequency radio waves (3 to 30 *MHz*). The region where the lighter atomic ions ( $H^+$  and  $He^+$ ) dominate.

# Importance of Layers



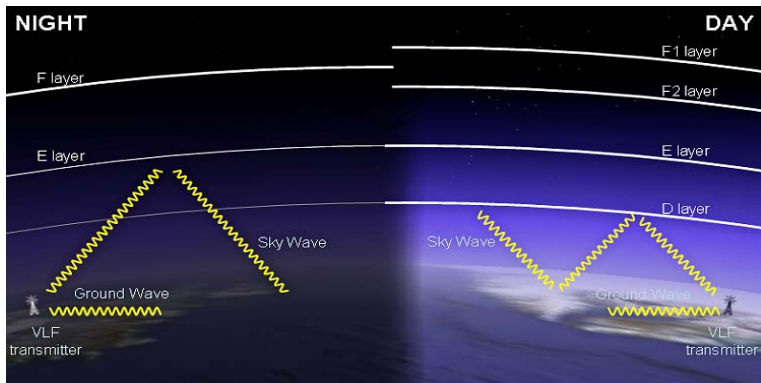




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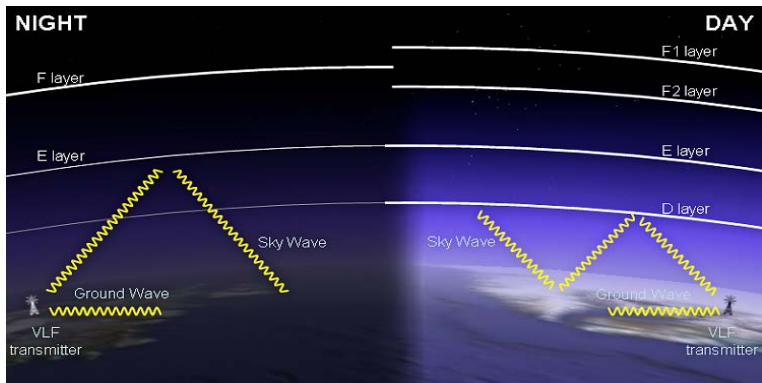


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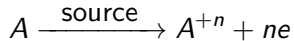
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## F:

- 150 – 500 km
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- HF are reflected
- species O, N, H

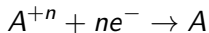
A process in which an atom or molecule loses at least one electron or more,



Several factors contribute to the ionization in the ionosphere:

- ① **Photoionization:** This daytime process occurs when the Sun's ultraviolet (UV) and X-ray radiation strips electrons from neutral atoms and molecules, creating a region of charged particles known as plasma.
- ② **Cosmic Rays:** During the night, high-energy particles or clusters of particles, called cosmic rays, originating from the Sun or distant stars, can penetrate the Earth's atmosphere and cause secondary electron emission, leading to further ionization.
- ③ **Superthermal Electrons:** These are high-energy electrons that can travel long distances and contribute significantly to heating the lower and mid-latitude ionosphere.

An ion combines with a free electron (or more) and becomes a neutral atom, molecule, or a less ionized species.



In the Earth's ionosphere, the main recombination mechanisms involve the recombination of  $O^{2+}$  and  $NO^{+}$  ions with free electrons.

Dissociative recombination occurs when electrons attach to positively charged molecular ions, forming highly energetic molecules that can undergo further electron removal.

## Balance

The balance between ionization and recombination processes determines the amount of ionization present in the ionosphere.

## Main aim

To use the present to predict the future through the application of physical models.

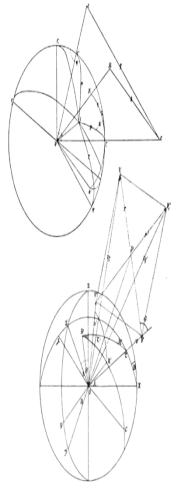
## Importance:

- To determine the densities of species crucial for life, such as ozone.
- To study variations caused by external disturbances like solar wind, magnetic storms, cyclones, and earthquakes.
- To ensure the functionality of satellites.
- To study space weather and its impact on our meteorological systems.
- To contribute to the prevention of future natural disasters.

# Boltzmann distribution

One of the greatest scientists who made significant contributions to **statistical mechanics**, which is:

- A pillar of modern physics.
- relates macroscopic parameters to microscopic ones.
- used in various fields, including medicine and stock markets.
- is based on probability theory.



Boltzmann statistics is based on fundamental concepts

## ① Phase Space

For a system of  $N$  particles, each particles has 6 degrees of freedom: position  $(x, y, z)$  noted  $\mathbf{r}$  and momentum  $(p_x, p_y, p_z)$  noted  $\mathbf{p}$ .

The set:  $(\mathbf{r}, \mathbf{p}) = (x, y, z, p_x, p_y, p_z)$  refers to **Phase Space**.

The elementary volume in the phase space is

$$d^3 r d^3 p = dx dy dz dp_x dp_y dp_z$$

## ② Indistinguishable particles:

The number of these particles is immense ( Avogadro's number), and all particles within the same type are identical. Consequently, it is impossible to track the movement of individual particles.



## 3 Density

The probability of a particle having coordinates  $(\mathbf{r}, \mathbf{p})$  is

$$dN = f(\mathbf{r}, \mathbf{p}, t) d^3\mathbf{r} d^3\mathbf{p}$$

where  $f((\mathbf{r}, \mathbf{p}), t)$  is a **probability density function** such as the number of particles in the elementary volume of coordinates  $(\mathbf{r}, \mathbf{p})$  is

$$dN = f(\mathbf{r}, \mathbf{p}, t) d^3\mathbf{r} d^3\mathbf{p}$$

the number of particles in a specific volume is

$$N = \int \int_{\text{Domain}} d^3\mathbf{p} d^3\mathbf{r} f(\mathbf{r}, \mathbf{p}, t)$$

- 4 Domain of the same volume have the same number of particles.
- 5 In the phase space the minimum volume is  $h^3$  (Planck const.)

Under a force  $\mathbf{F}$  at  $t$ , the change in a volume of coordinates  $(\mathbf{r}, \mathbf{p})$  after a time period  $\Delta t$  are  $\mathbf{r} + d\mathbf{p}$  and  $\mathbf{p} + d\mathbf{p}$  such as,

$$f\left(\mathbf{r} + \frac{\mathbf{p}}{m} \Delta t, \mathbf{p} + \mathbf{F} \Delta t, t + \Delta t\right) d^3\mathbf{r} d^3\mathbf{p} = f(\mathbf{r}, \mathbf{p}, t) d^3\mathbf{r} d^3\mathbf{p}$$

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The total number of particle is conserved

Taylor development

$$f\left(\mathbf{r} + \frac{\mathbf{p}}{m} \Delta t, \mathbf{p} + \mathbf{F} \Delta t, t + \Delta t\right) d^3\mathbf{r} d^3\mathbf{p} - f(\mathbf{r}, \mathbf{p}, t) d^3\mathbf{r} d^3\mathbf{p} = \Delta f d^3\mathbf{r} d^3\mathbf{p}$$

There are two possibilities

$$\frac{df}{dt} = 0, \quad \text{or} \quad \frac{df}{dt} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

Corresponds to

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \left( \frac{\partial f}{\partial t} \right)_{\text{coll}} \quad (*)$$

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For example by using zeroth-order moment (relative to  $V$ ), we multiply BE by  $v^0 = 1$  and the integrating over the phase space

$$\int_{-\infty}^{\infty} \left\{ \frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} \right\} d^3 r d^3 p = \int_{-\infty}^{\infty} \left( \frac{\partial f}{\partial t} \right)_{\text{coll}} d^3 r d^3 p$$

From the previous equation, when the collision term vanishes, one obtains, after some mathematical simplification,

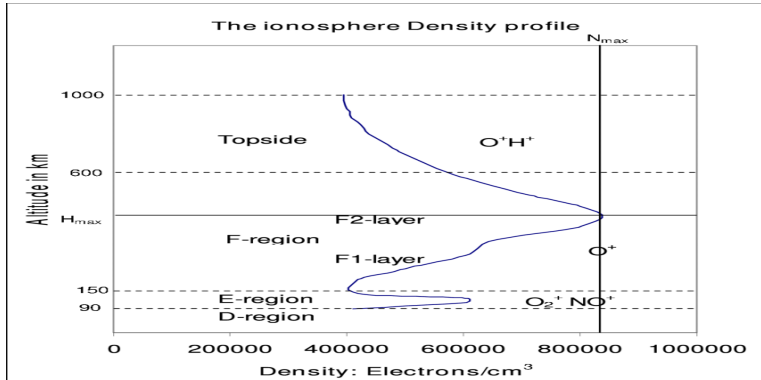
$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0$$

We are seeking the density, but this equation introduces a new unknown variable,  $v$ . Therefore, we need an additional equation to account for the first-order momentum term ( $mv$ ). Unfortunately, this approach can lead to an infinite series of equations if not addressed carefully.

The set of fluid equations must be closed by an additional equation, such as the ideal gas equation or an equation describing energy transfer processes,...

# What we want?

For example, let's examine the density profile versus altitude in the Earth's ionosphere.



Depending on our desired application (such as wave propagation or energy transfer), we can focus on a specific region of the ionosphere.



In principle, solving equations (\*) and (\*\*) allows us to determine the electron density. However, several factors need to be considered:

- 1 **Available data:** If the primary source of electrons is solar radiation (UV, EUV), the Boltzmann equations should be written in terms of the energy flux, denoted by  $\phi_e(\mathbf{r}, E, \omega, t)$ .

$$f_e(\mathbf{r}_e, \mathbf{v}_e, t) = \frac{m_e}{v^2} \phi_e(\mathbf{r}, E, \Omega, t) \quad (1)$$

To obtain

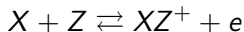
$$\begin{aligned} \frac{1}{v} \frac{\partial \phi_e(\mathbf{r}, E, \Omega, t)}{\partial t} + \frac{\mathbf{v}_s}{v} \nabla_r (\phi_e(\mathbf{r}, E, \Omega, t)) - n_e \frac{\partial}{\partial E} (L(E) \cdot \phi_e(\mathbf{r}, E, \Omega, t)) \\ = \frac{1}{v} Q(\mathbf{r}, E, \Omega, t) \end{aligned} \quad (2)$$

In this equation we have to consider all input/output

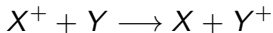
- 2 High energy electrons flowing from higher layers called **suprathermal electron** and all other electron sources have to be included in electron "budget",

$$P_\nu = n(z) \int_0^\infty \sigma_\nu(E) \phi(E) dE \quad (3)$$

- 3 The consideration of electron sink/source terms depends on the altitude range of the studied ionospheric region.



but the species  $X$  can also be involved in chemical reactions



So we have to find  $I_{X+Y} \cdot n_Y = K_{X+Y} \cdot n_{X^+} \cdot n_Y$

Obtaining **reaction** rates (reaction cross sections) is challenging.

# Model validation

Each species requires a continuity equation, and the momentum equation is also necessary if transport is considered.

## Code

Solve numerically the set of Boltzmann and fluid equations.

**We need inputs to start the Code**

Model Validation:

- **Comparison with available data:**
  - Satellite web data repositories: Data should be downloaded from reliable and credible sources.
  - Data formatting: The obtained data needs to be converted to a format compatible with the model.
- Comparison with established reference models (e.g., IRI, MSIS): This step evaluates how well the model's predictions align with the outcomes of trusted and widely used models.

The data on which this article is based are available in

[https://ccmc.gsfc.nasa.gov/modelweb/models/iri2016\\_vitmo.php](https://ccmc.gsfc.nasa.gov/modelweb/models/iri2016_vitmo.php)