Lecture (2) Electromagnetism
Simple Story

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Introduction

Text Books:

- ***College Physics, Serway / Vuille, Eight Edition, Chapters 15-18.***
- **Sadiku, *Elements of Electromagnetics, Oxford University.***
- ***Griffiths, Introduction to Electrodynamics, Prentice Hall.***
- ***Jackson, Classical Electrodynamics*, New York: John Wiley & Sons.**
- ***Open sources: MIT open courses, ….***

Keywords

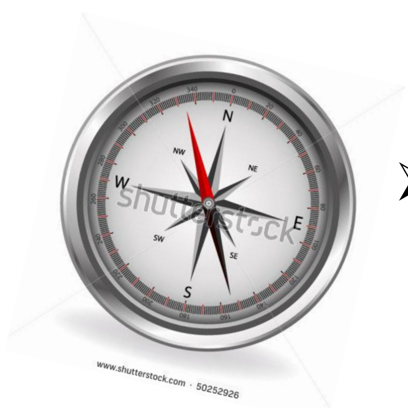
- ***Magnetic Force***
- ***Magnetic Field***
- ***Magnetic Potential***
- ***Biot-Savart's Laws***
- ***Ampere's law***
- ***Maxwell's law in Magnetostatic***

Compass

- A compass is a navigational instrument that measures directions in a frame of reference that is stationary relative to the surface of the Earth.

- **There are different types of compass:**

- the magnetic compass contains a magnet that interacts with the earth's magnetic field and aligns itself to point to the magnetic poles;



- the gyro compass (sometimes spelled with a hyphen, or as one word) contains a rapidly spinning wheel whose rotation interacts dynamically with the rotation of the earth.

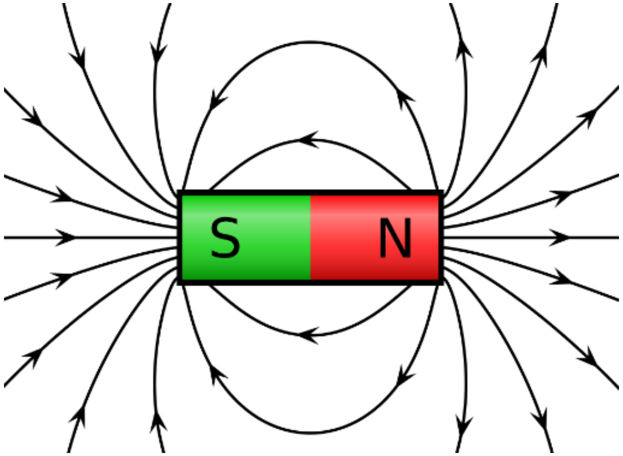
Basic facts of Magnetism

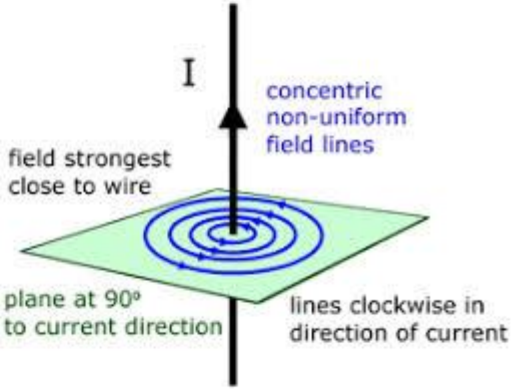
- ***<u>Oersted</u>*** discovered that a compass needle responded to the current in a loop of wire
- ***<u>Ampere</u>*** deduced the law for how a magnetic field is produced by the current in a wire
- magnetic field lines are always closed loops – no isolated magnetic poles, always have north and south
- <u>permanent magnets</u>: the currents are *atomic currents* – due to electrons spinning in atoms- these currents are always there
- <u>electromagnets:</u> the currents flow through wires and require a power source, e.g. a battery

Magnetic Flux

- Magnetic flux associated with a magnetic field is defined in a way similar to electric flux

$$\psi = \int \mathbf{B} \cdot d\mathbf{S}$$



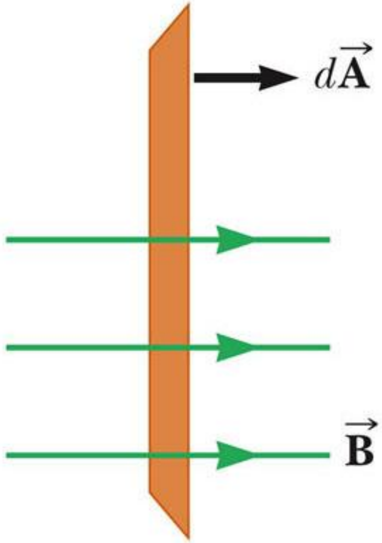


SI unit of flux: Weber

Wilhelm Eduard Weber
1804 – 1891

Magnetic Flux

- **For a flat surface with an area A in a uniform magnetic field, the flux is (θ is the angle between B and the normal to the plane):**



$$\psi = \int \mathbf{B} \cdot d\mathbf{S}$$

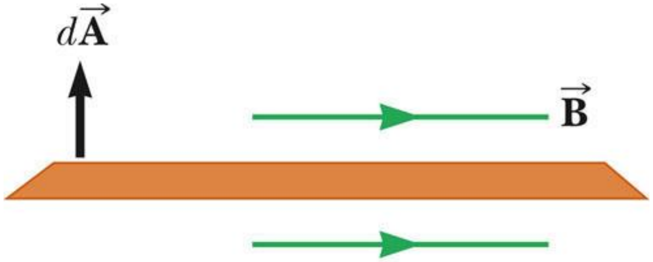
$$\psi = BS\cos(\theta) = BS$$

- **When the field is** **perpendicular**
  and the flux crosses the surface
- **When θ = 180 and the flux crosses the surface is negative.**

Magnetic Flux

- **When the field is parallel to the plane, θ = 90° and the flux crosses the surface is zero**

$$\psi = \int \mathbf{B} \cdot d\mathbf{S}$$



$$\psi = BS\cos(90) = 0$$

- **NOTE: the integration across a closed surface is zero, there is no magnetic monopole.**

$$\psi = \oint \mathbf{B} \cdot d\mathbf{S} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

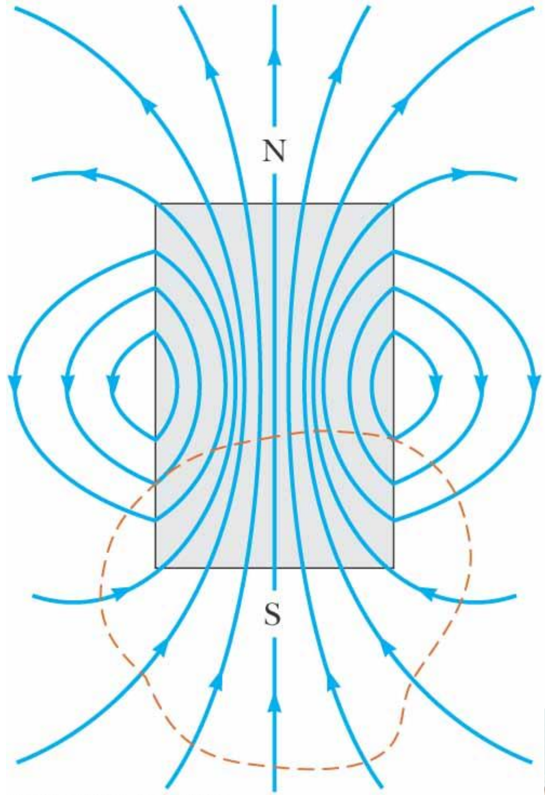
Magnetic flux

What is that magnetic flux through this surface?

A. Positive

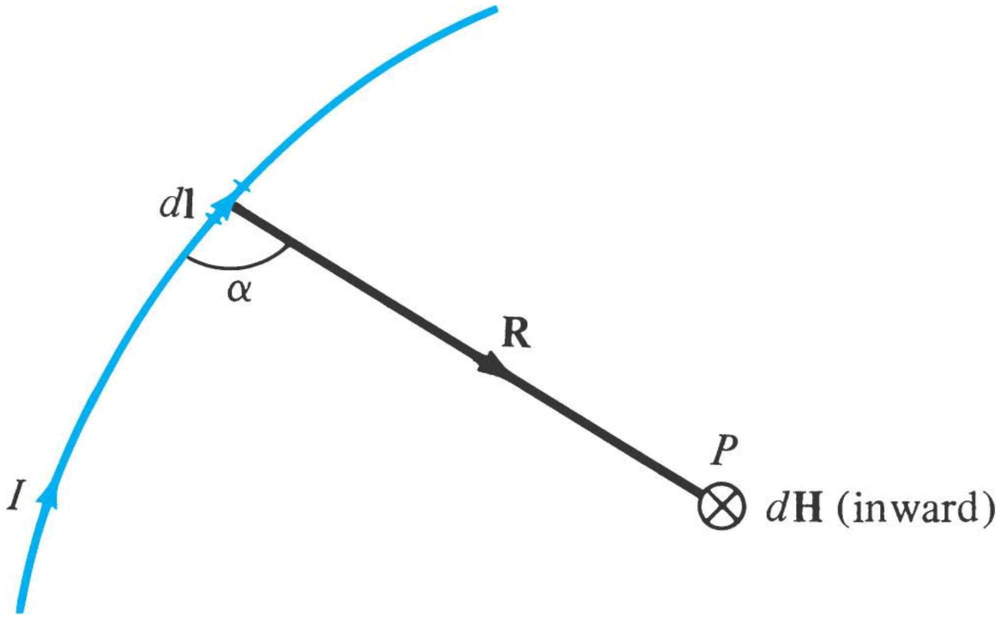
B. Negative

C. Zero



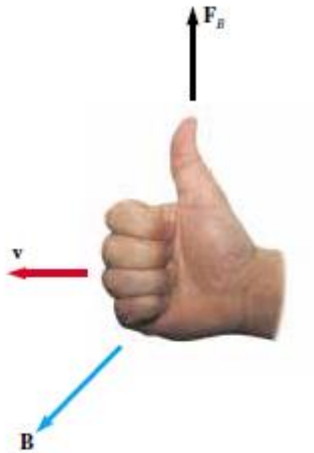
Source of a Magnetic Field

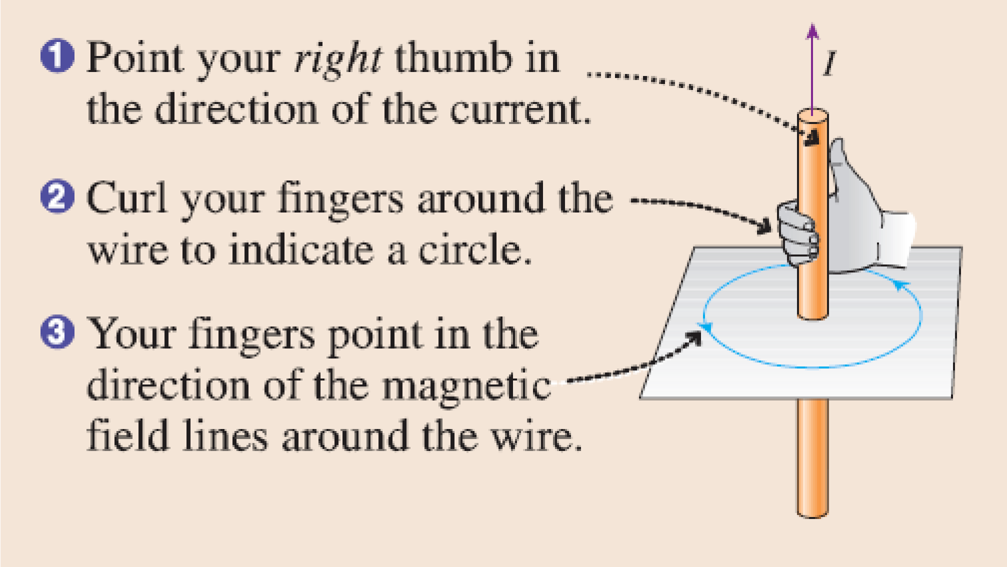
Biot-Savart's Law



$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I dl \times \mathbf{a}_r}{r^2}$$

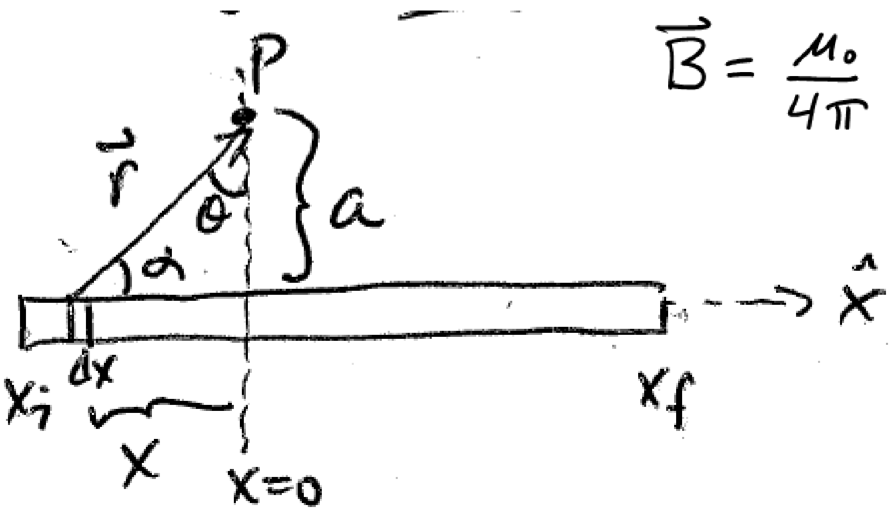
Summary of three Right Hand Rules:





Applying the Biot-Savart's Law

- **B-field for straight wire segment with current in x- direction:**

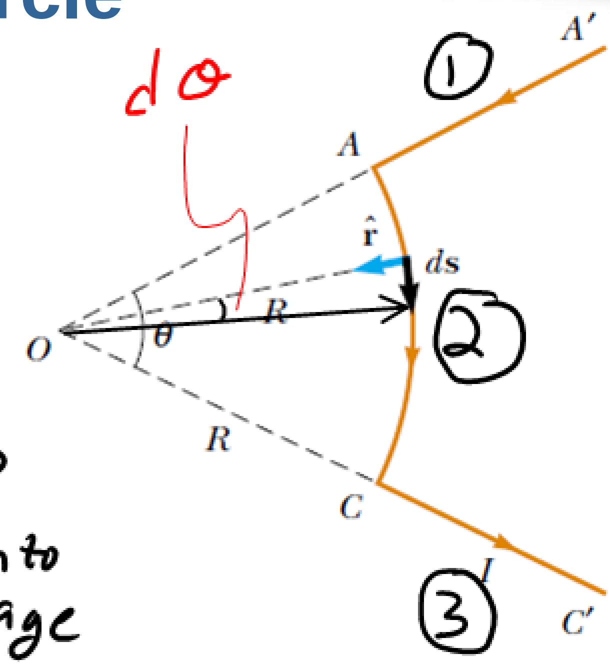


$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{x_f}{\sqrt{x_f^2 + a^2}} - \frac{x_i}{\sqrt{x_i^2 + a^2}}\right]\hat{z}$$

- **for an infinitely long straight wire:**

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{a}\hat{z}$$

Circular arc → Circle



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \hat{r}}{r^2}$$

For ① + ③, $d\vec{s} \parallel \hat{r} \Rightarrow d\vec{s} \times \hat{r} = 0$

For ②, $d\vec{s} \perp \hat{r} \Rightarrow d\vec{s} \times \hat{r} = ds$ into page

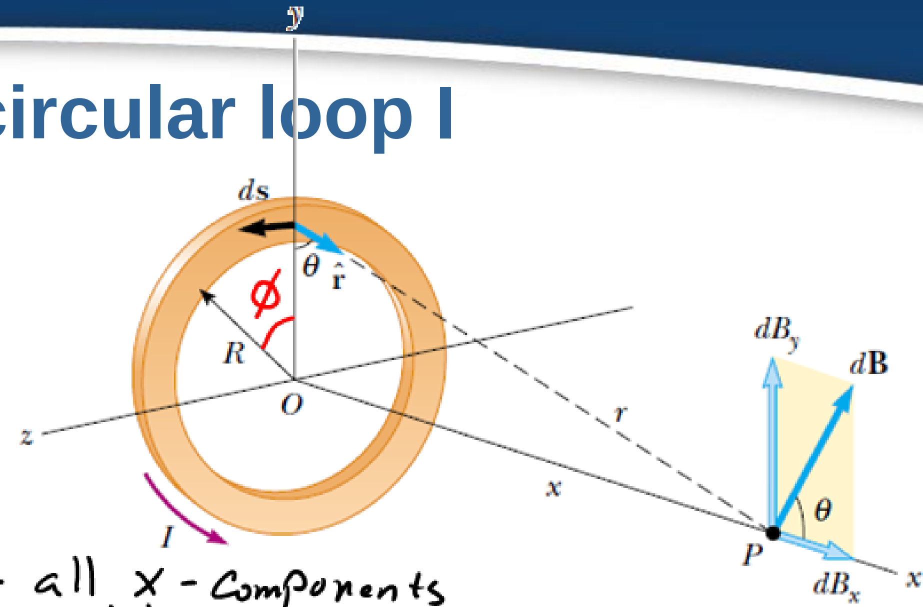
$$ds = a\,d\theta$$

$$|\vec{B}| = \int_{②} |d\vec{B}| = \frac{\mu_0}{4\pi} I \int_0^{\theta} \frac{a\,d\theta}{a^2} = \frac{\mu_0}{4\pi} \frac{I}{a}\theta$$

For Full Circle: $\theta \to 2\pi$

$$\vec{B}_{center} = \frac{\mu_0 I}{2a} \text{ into page}$$

On-axis of circular loop I



$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

When we integrate, $B_y \to 0$ & all x-components add

Let θ be angle between $d\vec{B}$ & $\hat{x}$:

$$|d\vec{B}| = d\vec{B}\cdot\hat{x} = |dB|\cos\theta$$

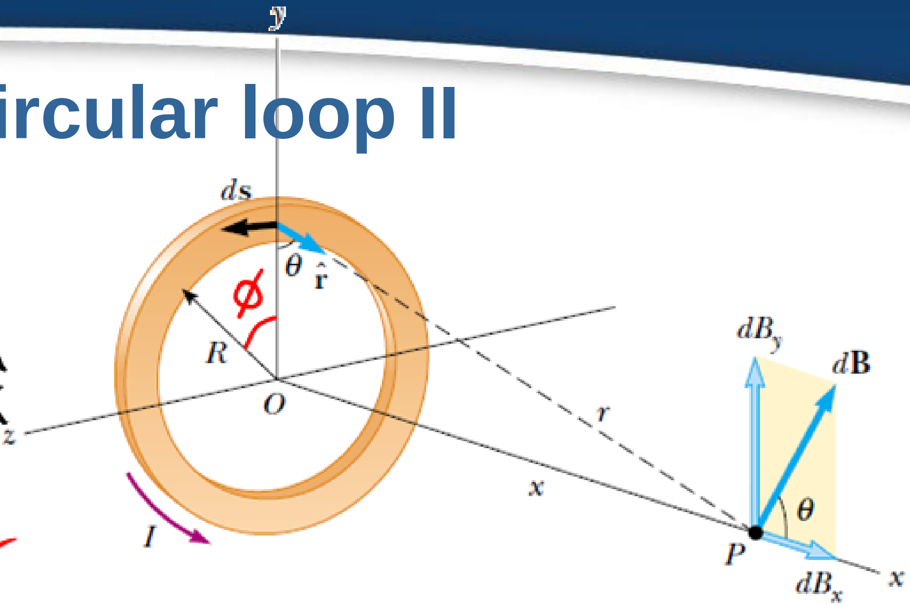
$$\cos\theta = \frac{R}{r}, \quad r = \sqrt{x^2 + R^2}, \quad d\vec{s} \perp \hat{r}$$

$$\Rightarrow |d\vec{s}| = R\, d\phi$$

$$|\vec{B}| = \int |dB| = \frac{\mu_0}{4\pi} I \int \frac{R\,\overbrace{d\phi}^{2\pi}}{r^2}\,\frac{R}{r}$$

$$\vec{B} = \frac{\mu_0}{2} I \frac{R^2}{(x^2+R^2)^{3/2}}\,\hat{x}$$

On-axis of circular loop II



$$\vec{B} = \frac{\mu_0}{2} I \frac{R^2}{(x^2+R^2)^{3/2}} \hat{x}$$

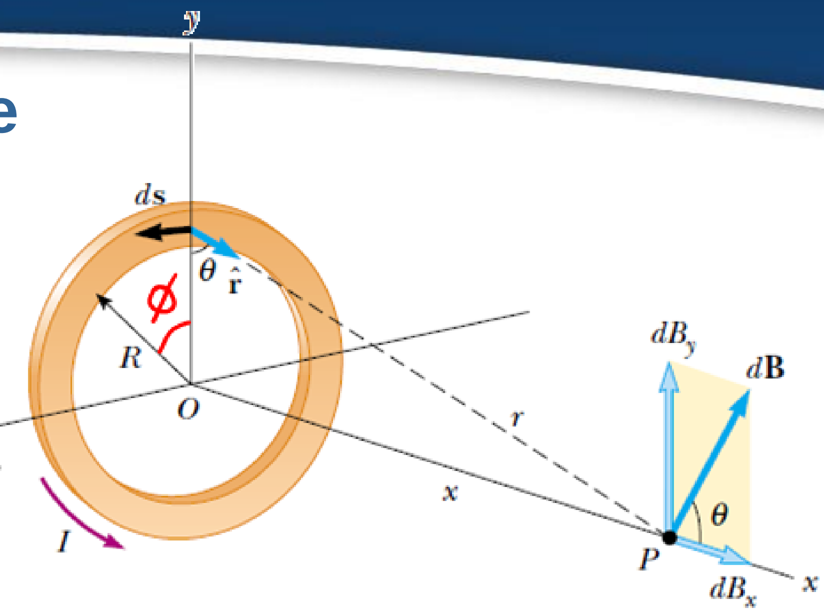
$$\lim_{x \to 0} \vec{B} = \frac{\mu_0 I}{2R} \hat{x} \checkmark$$

Let $x \gg R$, & $(x^2+R^2)^{3/2} = x^3 \underbrace{\left(1+\frac{R^2}{x^2}\right)^{3/2}}_{\to 1}$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} 2 \frac{\overbrace{I \pi R^2}^{\equiv \vec{\mu} = I\vec{A}}}{x^3} \hat{x}$$

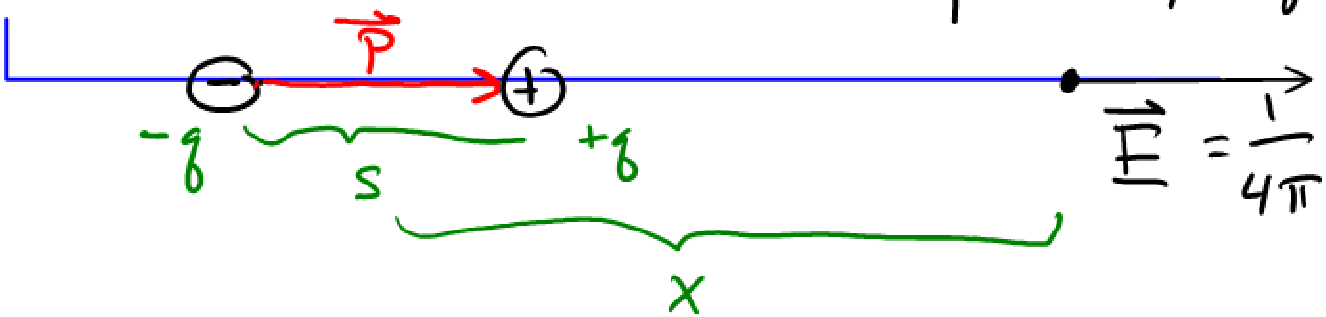
$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{x^3} \quad \text{on axis}$$

Magnetic dipole



$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{x^3} \quad \text{on axis}$$

Looks Like Electric dipole: $\vec{p} \equiv q\vec{s}$



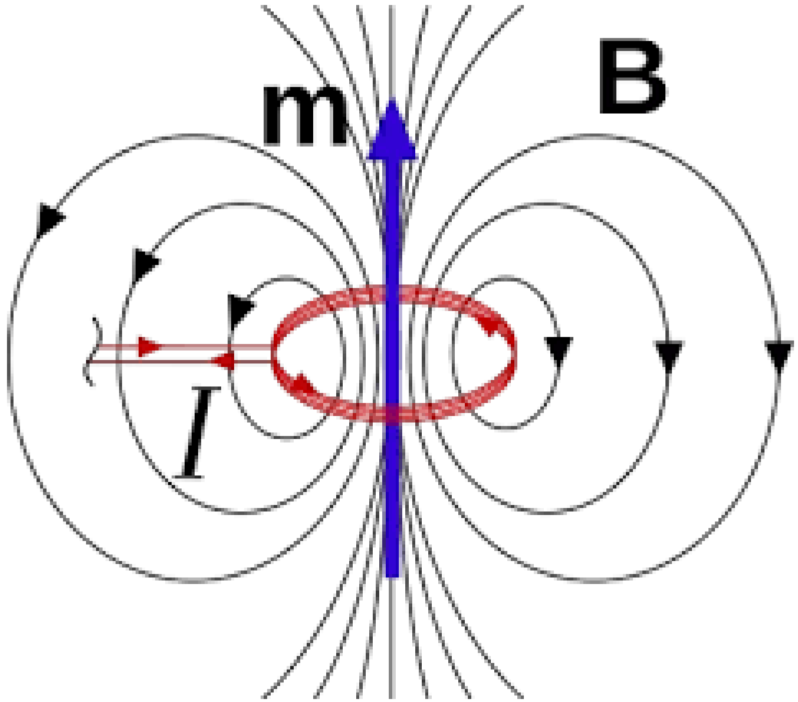
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{x^3}$$

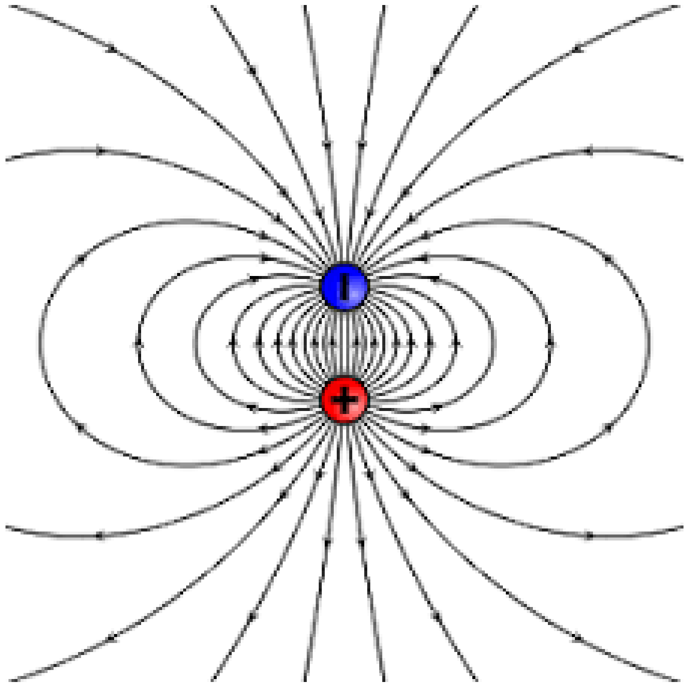
Also Recall, $\vec{\tau}_B = \vec{\mu} \times \vec{B}$ $\qquad$ $\vec{\tau}_E = \vec{p} \times \vec{E}$

$\qquad\qquad\qquad U_B = -\vec{\mu} \cdot \vec{B}$ $\qquad$ $U_E = -\vec{p} \cdot \vec{E}$

Current Loops are "magnetic Dipoles"

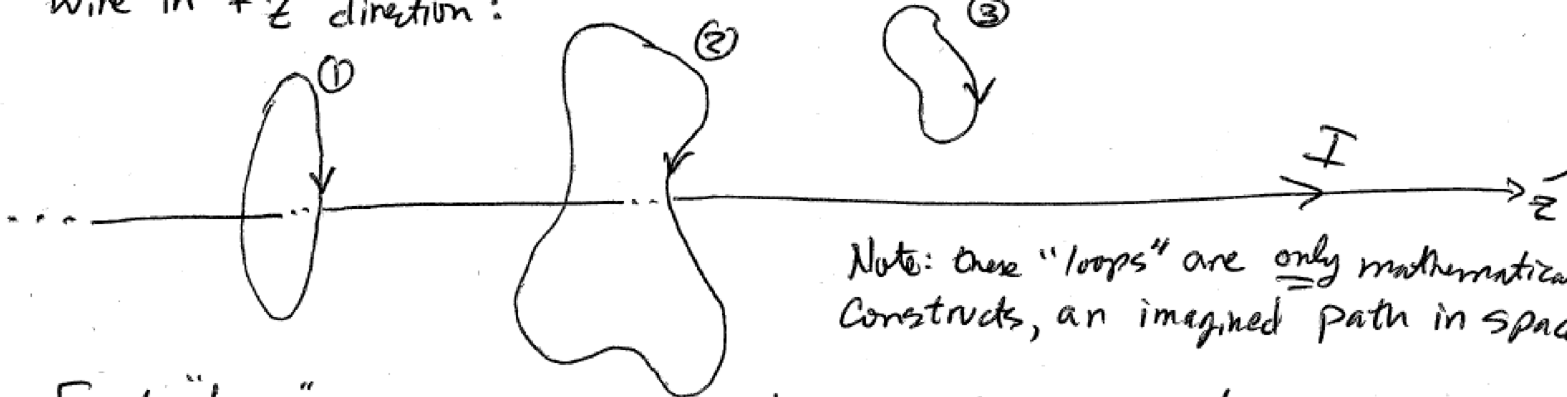
On-axis of circular loop III





Ampere's Law: An easier way to find B-fields

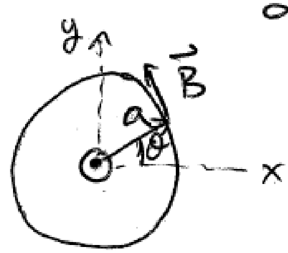
Consider the following case for an ∞-ite straight current carrying wire in $+\hat{z}$ direction:



Note: these "loops" are only mathematical constructs, an imagined path in space

Each "loop" is in the xy-plane. We will evaluate $\int \vec{B}\cdot d\vec{s}$ around the closed loops for case ①, ②, & ③ for an ∞-ite straight current carrying wire

①



$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{\theta}\,,\quad d\vec{s} = \underbrace{r}_{a}\, d\theta\,\hat{\theta} + \underbrace{dr}_{0,\ \text{constant radius}}\hat{r}$$

$$\oint \vec{B}\cdot d\vec{s} = \underbrace{\oint}_{2\pi} \frac{\mu_0 I}{2\pi a}\left[a\,d\theta\,\underbrace{\hat{\theta}\cdot\hat{\theta}}_{1} + \underbrace{\hat{\theta}\cdot\hat{r}}_{0}\,\underbrace{dr}_{0}\right]$$

$$\Rightarrow \oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

Ampere's Law: An easier way to find B-fields 1

Ampere's Law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\mathrm{enc}}$$

$$\oint (\nabla \times \mathbf{B}).d\mathbf{S} = \oint \mathbf{B} \cdot d\mathbf{l}$$

$$(\nabla \times \mathbf{B}) = \mu_0 \mathbf{J}$$

Use Ampere's Law to find B-field from current

1. **Current that does not go through "Amperian Loop" does not contribute to the integral**

2. **Current through is the "net" current through loop**

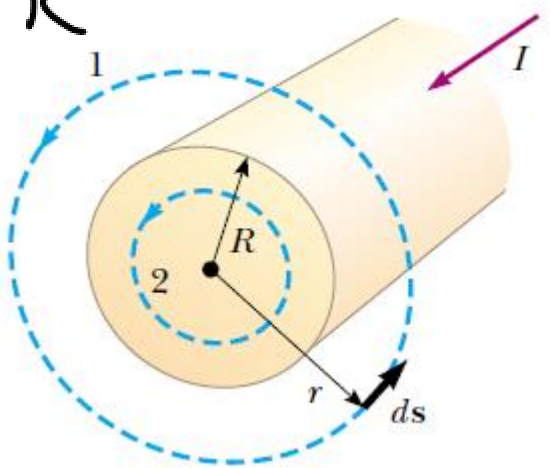
3. **Try to choose loops where B-field is either parallel or perpendicular to dl, the Amperian loop.**

Ampere's Law: Example, Finite size infinite wire

Calculate the B-field everywhere from a finite size, straight, infinite wire with uniform current.

Case I: outside of wire, $r \geq R$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$



$I_{enc} = I$, total current in wire

$\vec{B} \parallel d\vec{s}$

$$\vec{B} \cdot d\vec{s} = B\,ds = B\,r\,d\theta$$

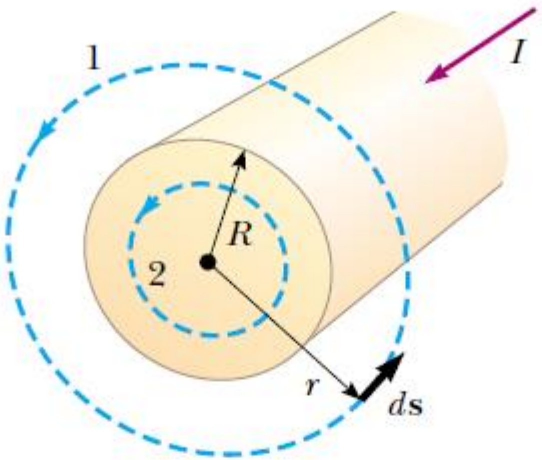
$$\oint \vec{B} \cdot d\vec{s} = \int_0^{2\pi} B\,r\,d\theta = \underbrace{2\pi r}_{\text{Circumference}} B = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

direction from RHR

Ampere's Law: Example, Finite size infinite wire

Calculate the B-field everywhere from a finite size, straight, infinite wire with uniform current.



Case II : $r \leq R$ inside wire

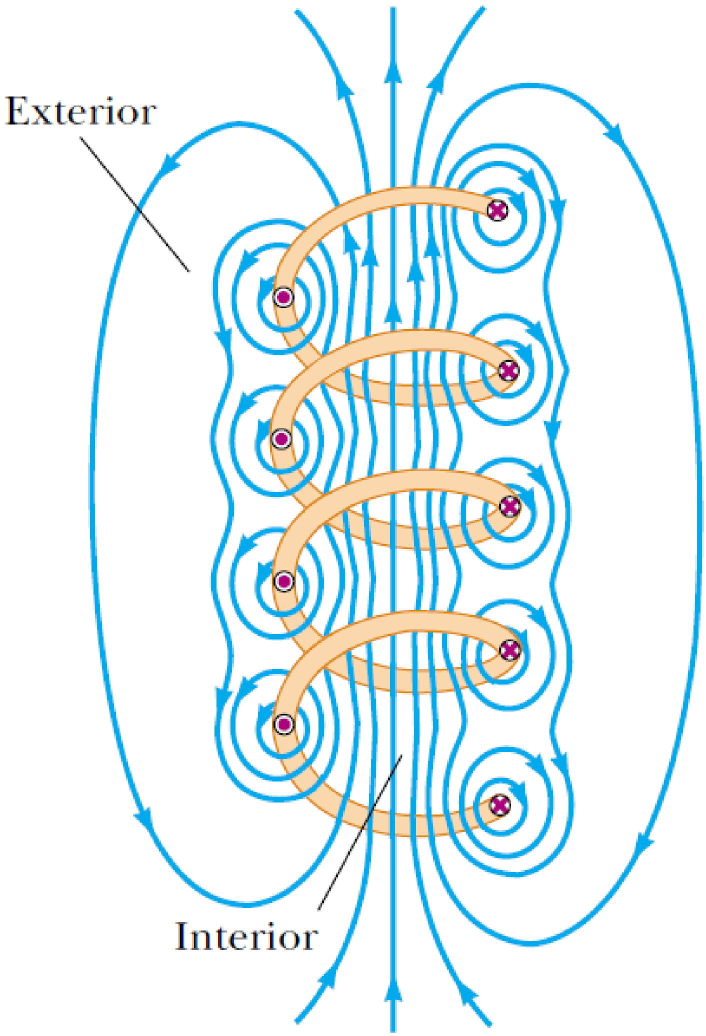
$$I_{enc} = \frac{\pi r^2}{\pi R^2} I$$

$$\oint \vec{B} \cdot d\vec{s} = \int_0^{2\pi} B r \, d\theta = 2\pi r B = \mu_0 I_{enc}$$

$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}$$

direction from RHR

Ampere's Law: Example, Infinitely long solenoid



As coils become more closely spaced, and the wires become thinner, and the length becomes much longer than the radius,

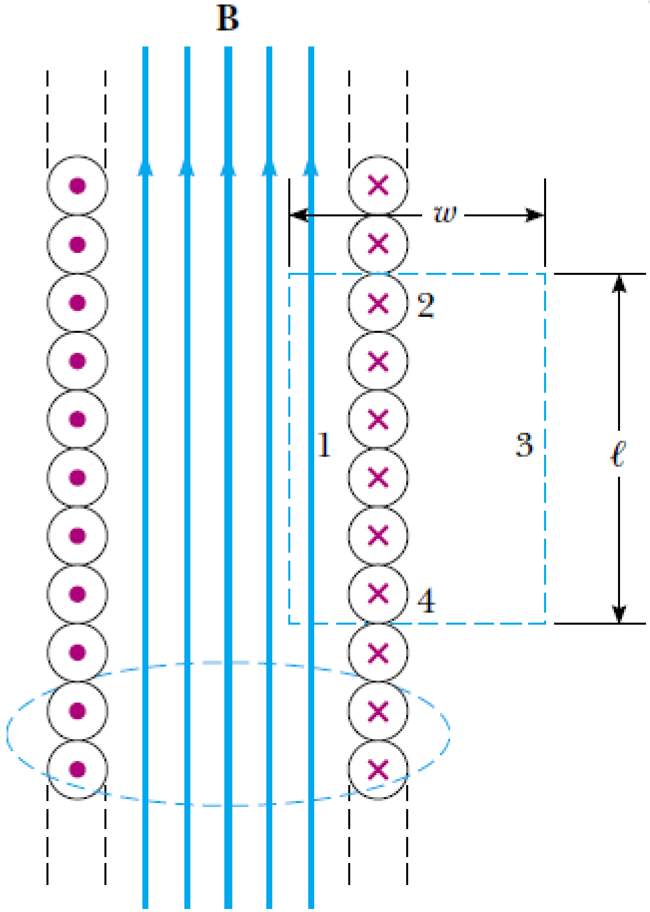
1. The B-field outside becomes very, very small (not at the ends, but away from sides)
2. The B-field inside points along the axial direction of the cylinder

Symmetry arguments for a sheet of current around a long cylinder:

1. Br=0 – time reversal + flipping cylinder, but time reversal would flip Br!
2. Azimuthal component? No, since if we choose Amperian loop perpendicular to axis, no current pierces it.
3. B=0 everywhere outside: any Amperian loop outside has zero current through it

Ampere's Law: Example, Infinitely long solenoid

Cross sectional view:



$$\oint \vec{B}\cdot d\vec{s} = \overbrace{\int_{1}\vec{B}\cdot d\vec{s}}^{\vec{B}\parallel d\vec{s},\ B\ell} + \overbrace{\int_{2}\vec{B}\cdot d\vec{s}}^{\vec{B}\perp d\vec{s},\ 0}$$

$$+ \underbrace{\int_{3}\vec{B}\cdot d\vec{s}}_{B=0\ \text{outside}} + \underbrace{\int_{4}\vec{B}\cdot d\vec{s}}_{\vec{B}\perp d\vec{s},\ 0}$$

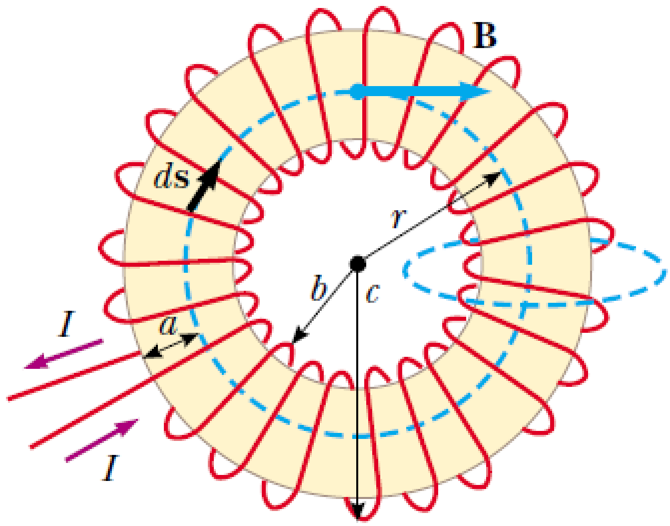
$I_{enc} = \frac{N}{L}\,\ell I$, N = # of turns
L = total length of Solenoid
I = Current in wire

$$B\ell = \mu_0\left(\frac{N}{L}\,\ell I\right) \Rightarrow B = \mu_0 n I,\ \text{RHR}$$

where n = # of turns per length

B is uniform inside solenoid!!

Ampere's Law: Example, Toroid



Solenoid bent in shape of donut.
B is circumferential.

$\therefore$ Draw amperian loop as circle inside torus

$\Rightarrow \vec{B} \parallel d\vec{s}$

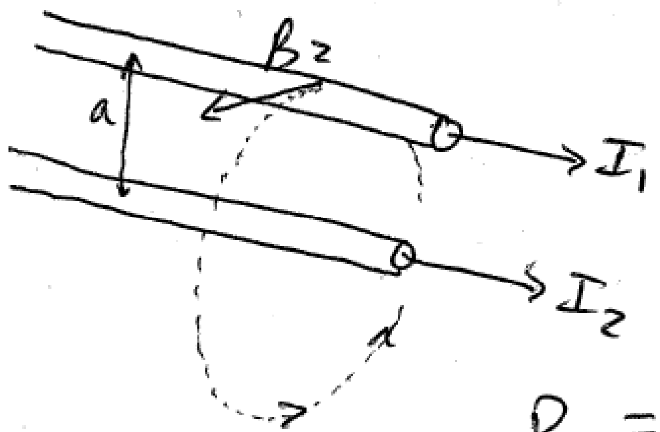
$$\oint \vec{B}\cdot d\vec{s} = B\cdot 2\pi r = \mu_0 (NI)$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 N I}{2\pi r}\,\hat{\theta}}$$

Force Between two parallel, straight current carrying wires:

- one wire produces a B-field @ location of other wire.

Consider 2 parallel wires carrying current:



I_2 produces B_2, & current I_1 feels a force. Let l_1 be the length of l_1 & wire 2 is infinitely long

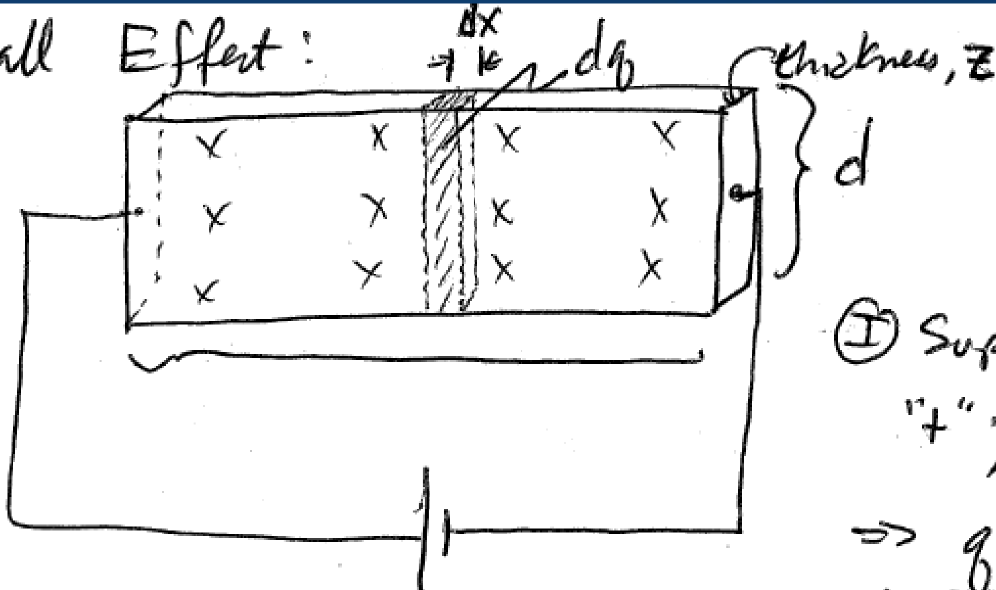
$$\Rightarrow \vec{F}_1 = I_1 \vec{l}_1 \times \vec{B}_2 = I_1 l_1 B_2 \text{ towards wire 2}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

$$\Rightarrow F_1 = I_1 l_1 \frac{\mu_0 I_2}{2\pi a} = \boxed{\frac{\mu_0}{2\pi a} I_1 I_2} \text{ towards wire 2}$$

Parallel currents attract, Opposite currents repel.

• Hall Effect:



Connect a Battery to Conducting Bar in magnetic field pointing into page.

(I) Suppose charge carriers are "+"; $\vec{V}_D \parallel \vec{I}$ (to the right)

$\Rightarrow q\vec{V}_D \times \vec{B}$ up $\Rightarrow$ "+" charge Carriers collect along top until $\vec{E}$ force equals $\vec{B}$ force:

$$q V_D B = q E_H \;, \quad E_H = \frac{\Delta V_H}{d} \quad \Rightarrow \Delta V_H = d\, V_D\, B$$

dq = amnt of charge in volume element ~~[illegible]~~, $A_\perp$ = cross sectional area = $d \cdot Z$

$dx \equiv V_D\, dt$ ∋ all dq will move out of volume element in time dt

$$dq = n\, q_0 \underbrace{(A_\perp\, dx)}_{\text{volume}} = n q_0\, d Z\, V_D\, dt \Rightarrow \frac{dq}{dt} = I = n q_0 d Z V_D$$

density of carriers; charge of carrier

or $\Delta V_H = B \cdot d \left[\frac{I}{n q_0 d Z}\right] = \boxed{\frac{I B}{n q_0 Z}}$

Maxwell's equations in statics

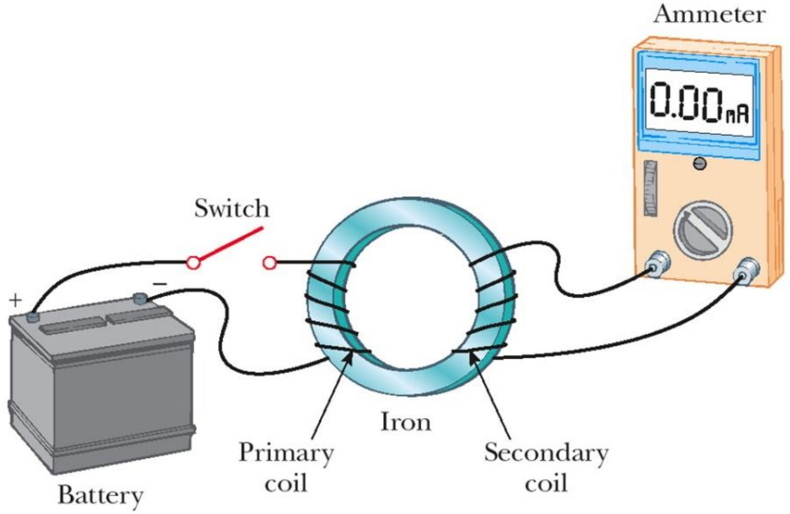
➢ ***Maxwell's equations***

Point Form	***Integral Form***
$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon}$	$Q/\epsilon = \oint \mathbf{E} \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{B} = 0$	$\psi = \oint \mathbf{B} \cdot d\mathbf{S} = 0$
$\nabla \times \mathbf{E} = 0$	$\oint \mathbf{E} \cdot \mathbf{dl} = 0$
$(\nabla \times \mathbf{B}) = \mu_0 \mathbf{J}$	$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$

Faraday's Experiment

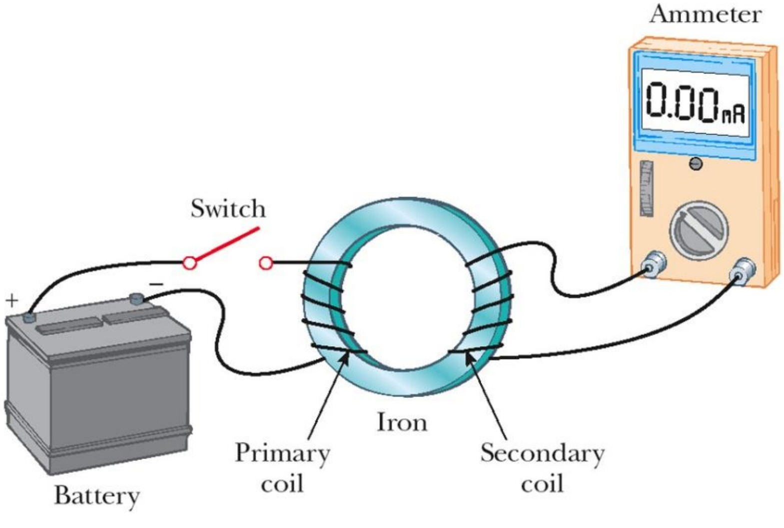
- **A primary coil is connected to a battery and a secondary coil is connected to an ammeter**
- **The purpose of the secondary circuit is to detect current that might be produced by a (changing) magnetic field**

- **When there is a steady current in the primary circuit, the ammeter reads zero**



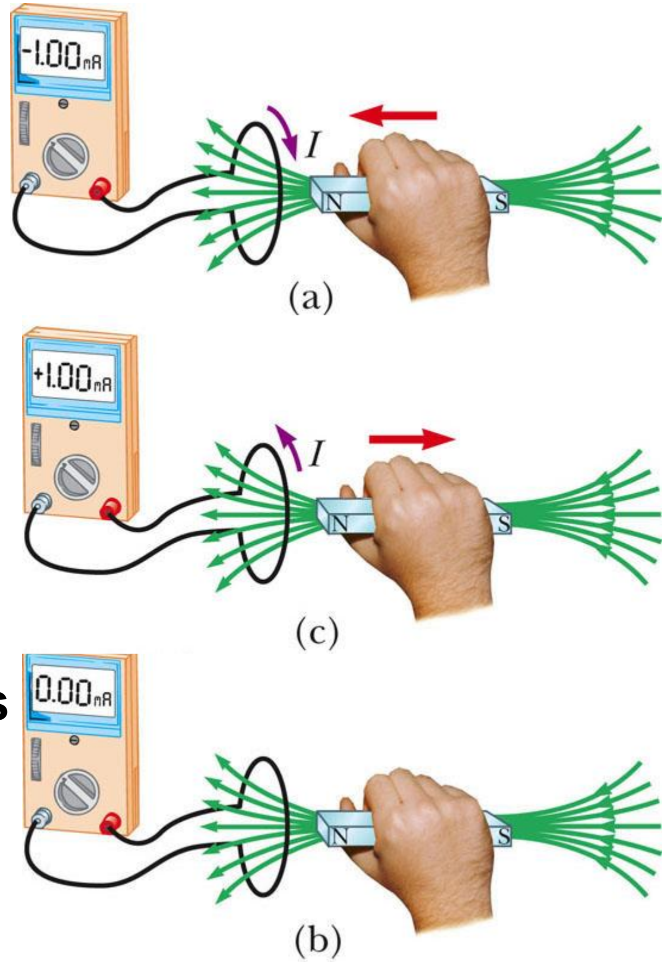
Faraday's Experiment

- When the switch is opened, the ammeter reads a current and then returns to zero
- When the switch is closed, the ammeter reads a current in the opposite direction and then returns to zero
- An induced emf is produced in the secondary circuit by the changing magnetic field



Electromagnetic Induction

- When a magnet **moves** toward a loop of wire, the ammeter shows the presence of a current
- When the magnet **moves** away from the loop, the ammeter shows a current in the opposite direction
- When the magnet is held **stationary**, there is no current
- If the loop is moved instead of the magnet, a current is also detected



Electromagnetic Induction

- **A current is set up in the circuit as long as there is relative motion between the magnet and the loop**
- **The current is called an induced current because is it produced by an induced emf**
- **Faraday's law of induction: the instantaneous emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit**
- **If the circuit consists of N loops, all of the same area, and if ψ is the flux through one loop, an emf is induced in every loop and Faraday's law becomes**

Faraday's Law and Lenz' Law

- **The negative sign in Faraday's Law is included to indicate the polarity of the induced emf, which is found by Lenz' Law*:***

- **The current caused by the induced emf travels in the direction that creates a magnetic field with flux opposing the change in the original flux through the circuit**

$$\epsilon = -N\frac{d\psi}{dt}$$

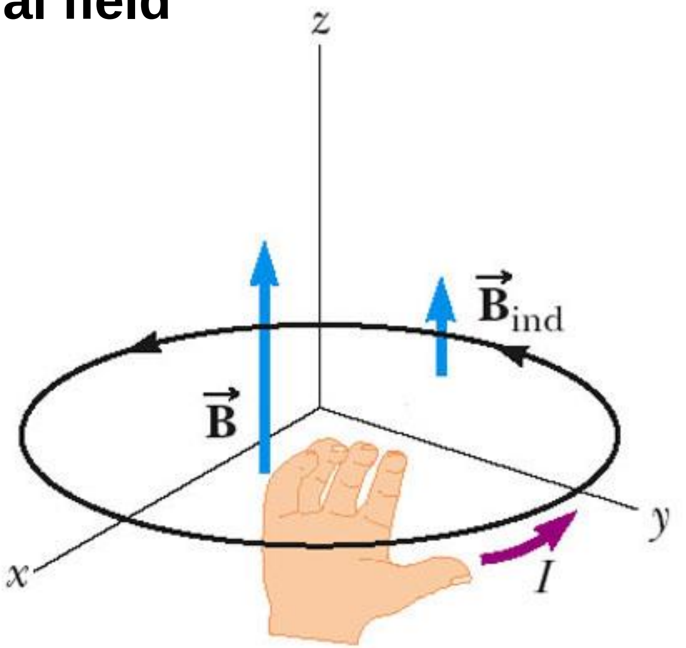
Heinrich Friedrich
Emil Lenz
1804 – 1865

Lenz' Law

- **The magnetic field, B, becomes smaller with time and this reduces the flux**

- **The induced current will produce an induced field, B_{ind}, in the same direction as the original field**

$$\epsilon = -N\frac{d\psi}{dt}$$

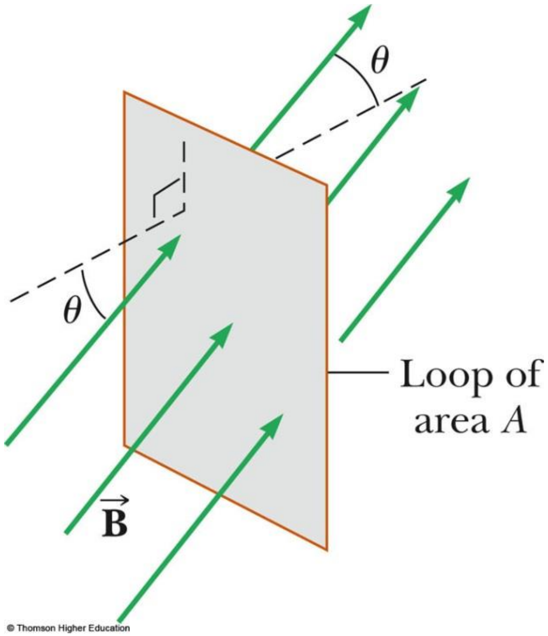


Faraday's Law

- **Assume a loop enclosing an area A lies in a uniform magnetic field**

- **Since** $\psi = BS\cos(\theta)$**, the change in the flux, can be produced by a change in B, A** or θ

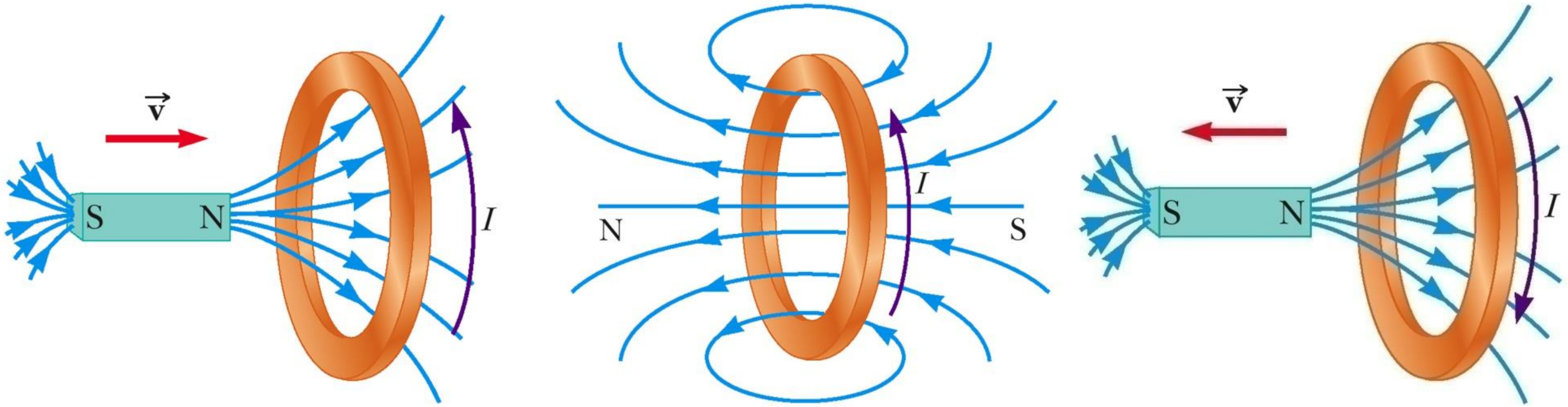
$$\epsilon = -N\frac{d\psi}{dt}$$



Lenz' Law – Moving Magnet

- **As the bar magnet is moved to the right toward a stationary loop of wire, the magnetic flux increases with time**

- **The induced current produces a flux to the left, so the current is in the direction shown**



Lenz' Law – Rotating Loop

- **Assume a loop with *N* turns, all of the same area rotating in a magnetic field**

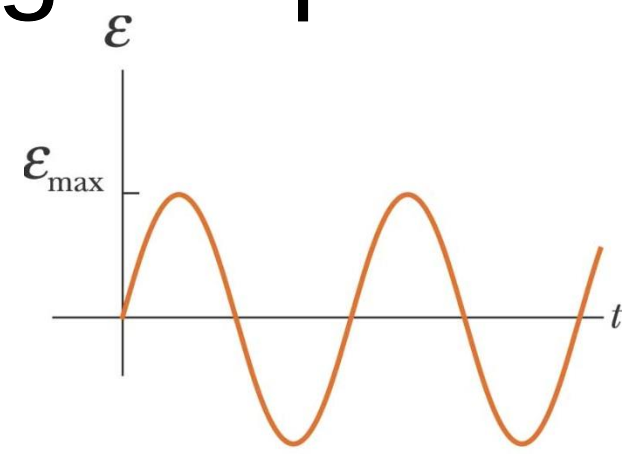
- **The flux through the loop at any time *t* is**

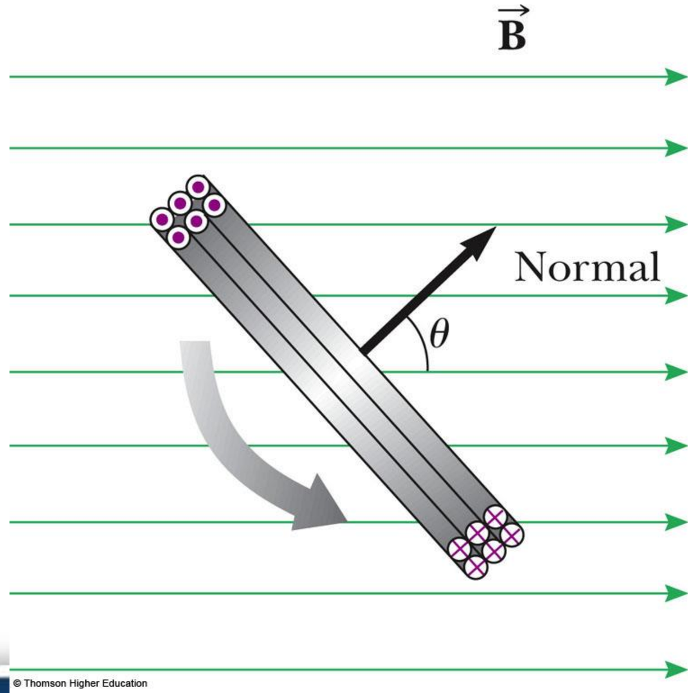
$$\psi = BS\cos(\theta) = BS\cos(\omega t)$$

- **The induced emf in the loop is**
- **This is sinusoidal, with ε_{max} = B *N* *S* ω**

$$\epsilon = -N\frac{d\psi}{dt}$$

$$\epsilon = B\,S\,N\,\omega\,\sin(\omega t)$$





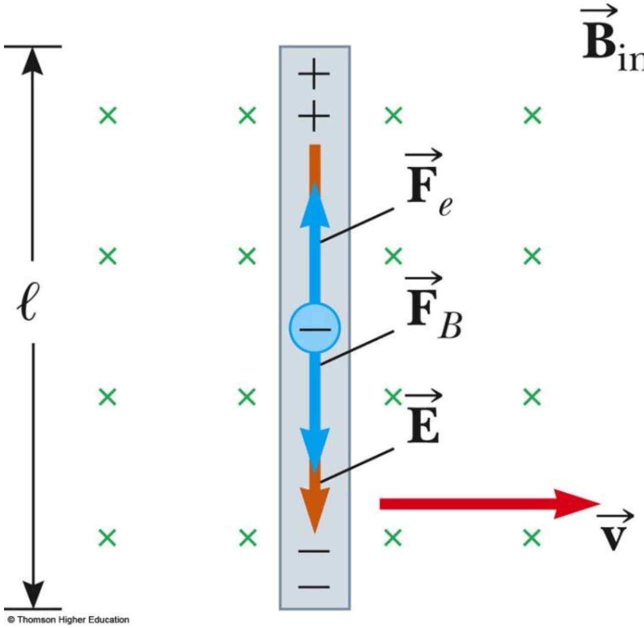
Motional emf

- **A straight conductor of length ℓ moves perpendicularly with constant velocity through a uniform field**
- **The electrons in the conductor experience**

  a magnetic force

$$\mathbf{F}_{\mathrm{B}} = e\mathbf{v} \times \mathbf{B}$$

- **The electrons tend to move to the lower end of the conductor**
- **As the negative charges accumulate at the base, a net positive charge exists at the upper end of the conductor**

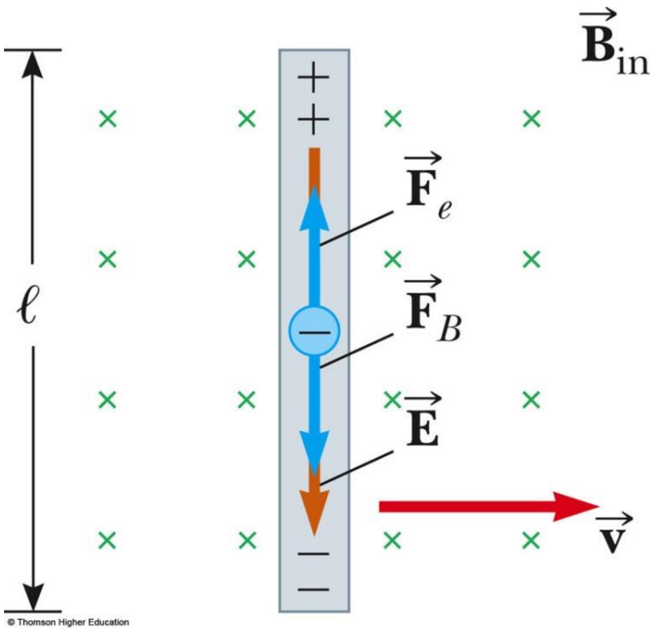


Motional emf

- **As a result of this charge separation, an electric field is produced in the conductor**

- **Charges build up at the ends of the conductor until the downward magnetic force is balanced by the upward electric force**

$$F_E = q\,E = q\,v\,B; \qquad E = v\,B;$$

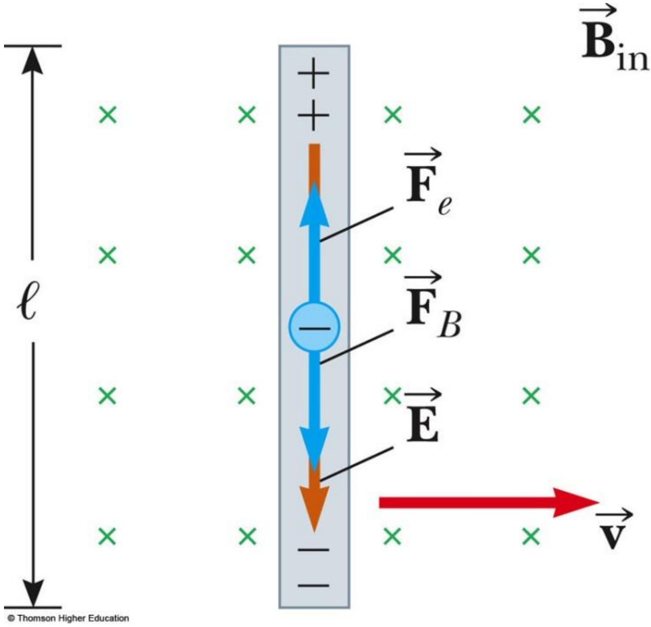


- **There is a potential difference between the upper and lower ends of the conductor**

Motional emf

- **The potential difference between the ends of the conductor (the upper end is at a higher potential than the lower end):**

$$\Delta V = E\,\ell = B\,\ell\,v$$



- **A potential difference is maintained across the conductor as long as there is motion through the field**

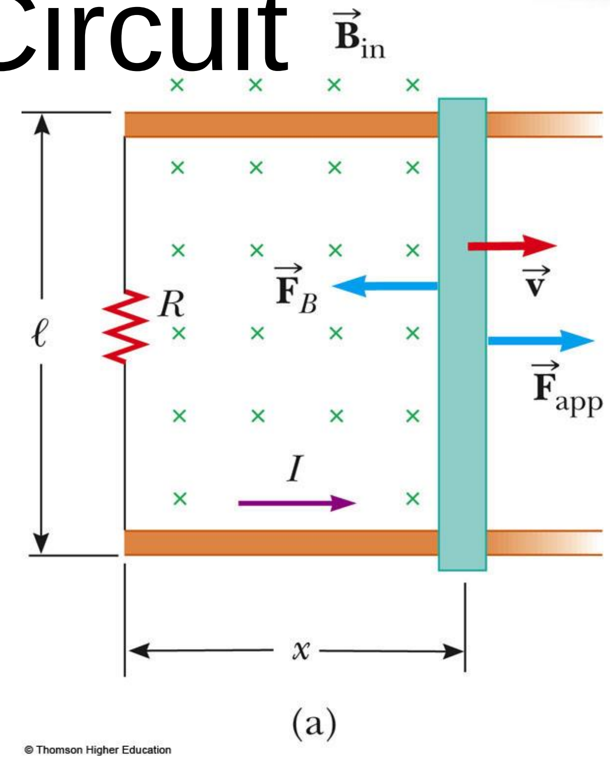
- **If the motion is reversed, the polarity of the potential difference is also reversed**

Motional emf in a Circuit

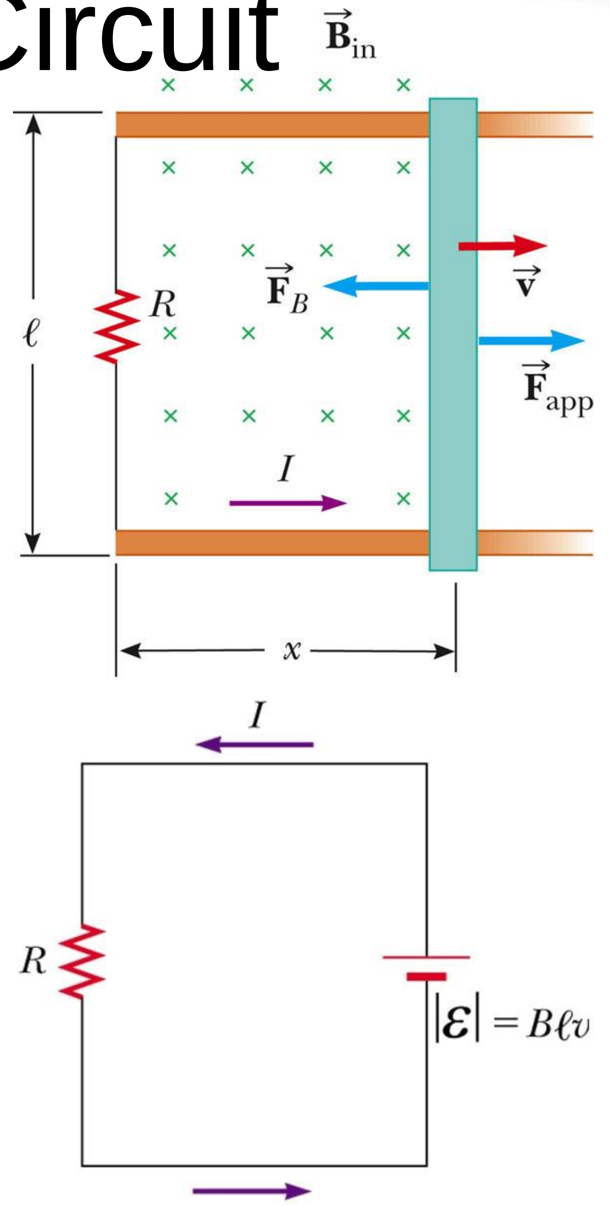
- **As the bar (with zero resistance) is pulled to the right with a constant velocity under the influence of an applied force, the free charges experience a magnetic force along the length of the bar**

- **This force sets up an induced current because the charges are free to move in the closed path**

- **The changing magnetic flux through the loop and the corresponding induced emf in the bar result from the change in area of the loop**



Motional emf in a Circuit



(b)

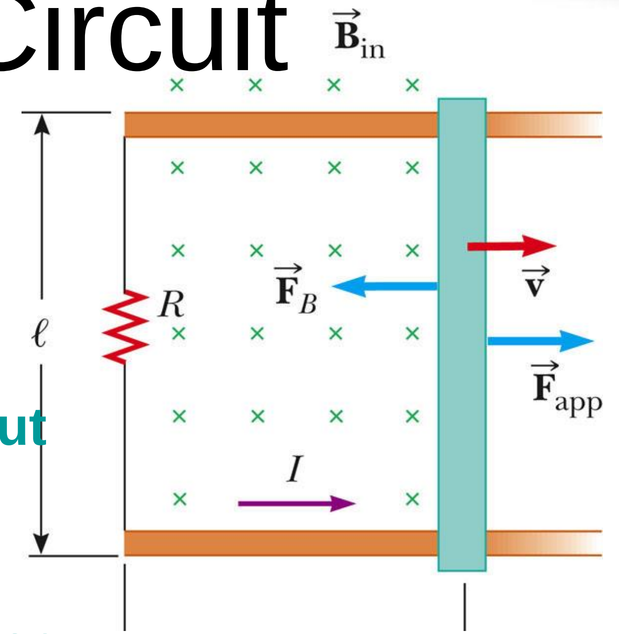
- **The induced, motional emf, acts like a battery in the circuit**

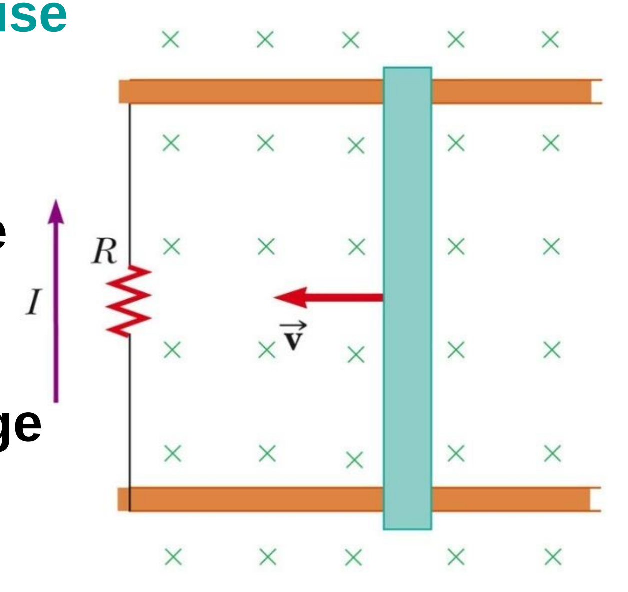
$$\boxed{|\varepsilon| = B\ell v} \quad and \quad \boxed{I = \frac{B\ell v}{R}}$$

- **As the bar moves to the right, the magnetic flux through the circuit increases with time because the area of the loop increases**
- **The induced current must be in a direction such that it opposes the change in the external magnetic flux (Lenz' Law)**

Motional emf in a Circuit

- **The flux due to the external field is increasing into the page**
- **The flux due to the induced current must be out of the page**
- **Therefore the current must be counterclockwise when the bar moves to the right**
- **If the bar is moving toward the left, the magnetic flux through the loop is decreasing with time – the induced current must be clockwise to produce its own flux into the page**





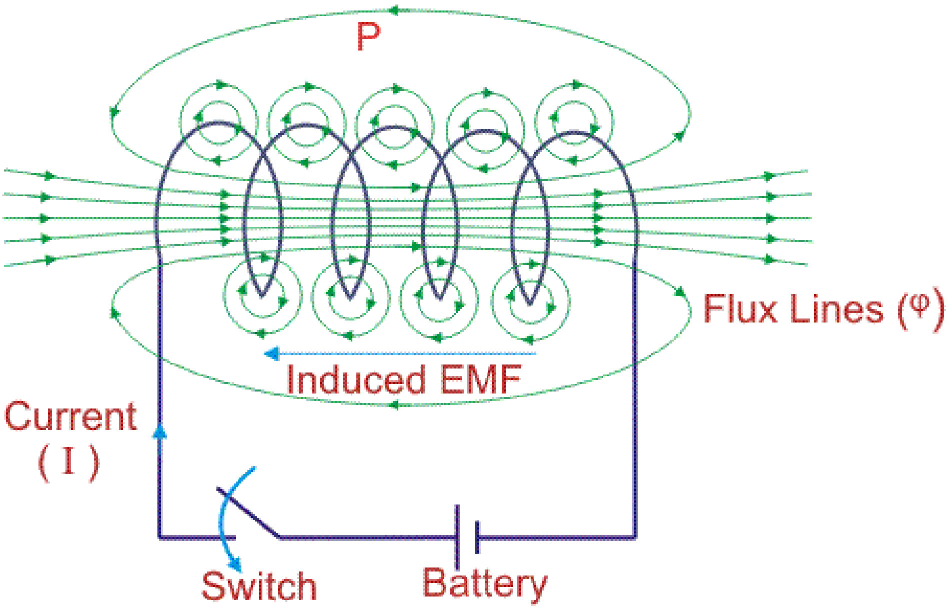
(b)

Self-inductance

- **Some terminology first:**
- **Use emf and current when they are caused by batteries or other sources**
- **Use induced emf and induced current when they are caused by changing magnetic fields**
- **It is important to distinguish between the two situations**

Self-inductance

- When the switch is closed,
  the current does not immediately reach its maximum value
- As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time
- This increasing flux creates an induced emf in the circuit
- The direction of the induced emf is opposite the direction of the emf of the battery



Self-inductance

- **The self-induced emf ε_L is always proportional to the time rate of change of the current. (The emf is proportional to the flux change, which is proportional to the field change, which is proportional to the current change)**

$$\varepsilon_L = -L\frac{dI}{dt} \qquad \epsilon = -N\frac{d\psi}{dt}$$

- **L: inductance of a coil (depends on geometric factors)**
- **The negative sign indicates that a changing current induces an emf in opposition to that change**
- **The SI unit of self-inductance: Henry**
- **1 H = 1 (V · s) / A**

Joseph Henry
1797 – 1878

Inductance of a Solenoid

- **Assume a uniformly wound solenoid having N turns and length ℓ (ℓ is much greater than the radius of the solenoid)**
- **The flux through each turn of area A is**

$$\Phi_B = BA = \mu_0 n I A = \mu_0 \frac{N}{l} I A$$

$$L = \frac{N\Phi_B}{I} = \frac{N\left(\mu_0 \frac{N}{l} \cancel{I} A\right)}{\cancel{I}} = \frac{\mu_0 N^2 A}{l}$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

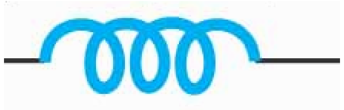
This shows that L depends on the geometry of the object

Inductor in a Circuit

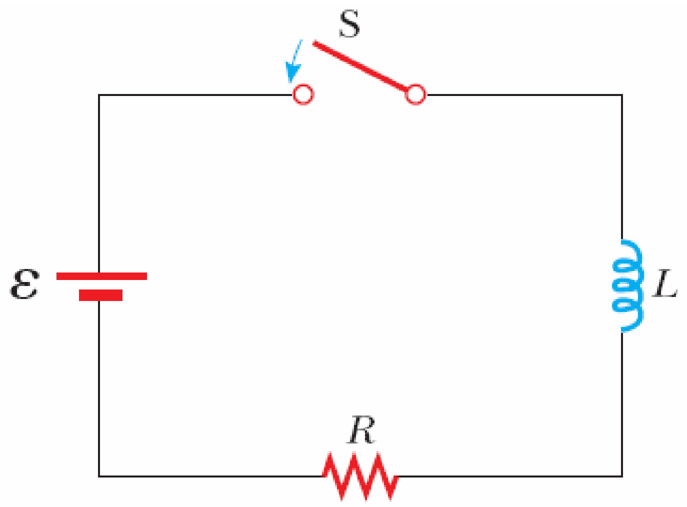
- **Inductance can be interpreted as a measure of opposition to the rate of change in the current (while resistance is a measure of opposition to the current)**
- **As a circuit is completed, the current begins to increase, but the inductor produces a back emf**
- **Thus the inductor in a circuit opposes changes in current in that circuit and attempts to keep the current the same way it was before the change**
- **As a result, inductor causes the circuit to be "sluggish" as it reacts to changes in the voltage: the current doesn't change from 0 to its maximum instantaneously**

RL Circuit

- A circuit element that has a large self-inductance is called an inductor
- The circuit symbol is



- We assume the self-inductance of the rest of the circuit is negligible compared to the inductor (However, in reality, even without a coil, a circuit will have some self-inductance
- When switch is closed (at time $t = 0$), the current begins to increase, and at the same time, a back emf is induced in the inductor that opposes the original increasing current



RL Circuit

Applying Kirchhoff's loop rule to the circuit in the clockwise direction gives

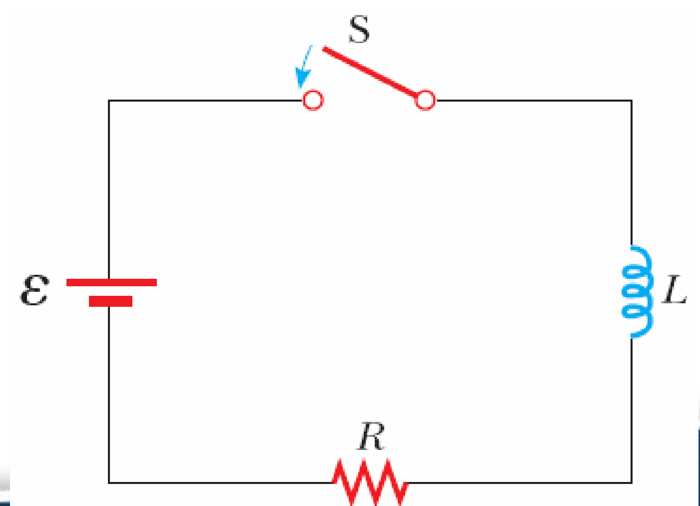
$$\varepsilon + \varepsilon_L - IR = 0 \qquad I(t=0) = 0$$

$$\varepsilon - L\frac{dI}{dt} - IR = 0 \qquad \frac{\varepsilon - IR}{L} = \frac{dI}{dt} \qquad \frac{dt}{L} = \frac{dI}{\varepsilon - IR}$$

$$\frac{Rdt}{L} = -\frac{d(\varepsilon - IR)}{\varepsilon - IR} \qquad -\frac{Rt}{L} = \ln(\varepsilon - IR) + \ln(Const)$$

$$e^{-\frac{Rt}{L}} = Const(\varepsilon - IR) = \frac{1}{\varepsilon}(\varepsilon - IR)$$

$$\boxed{I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)}$$



RL Circuit

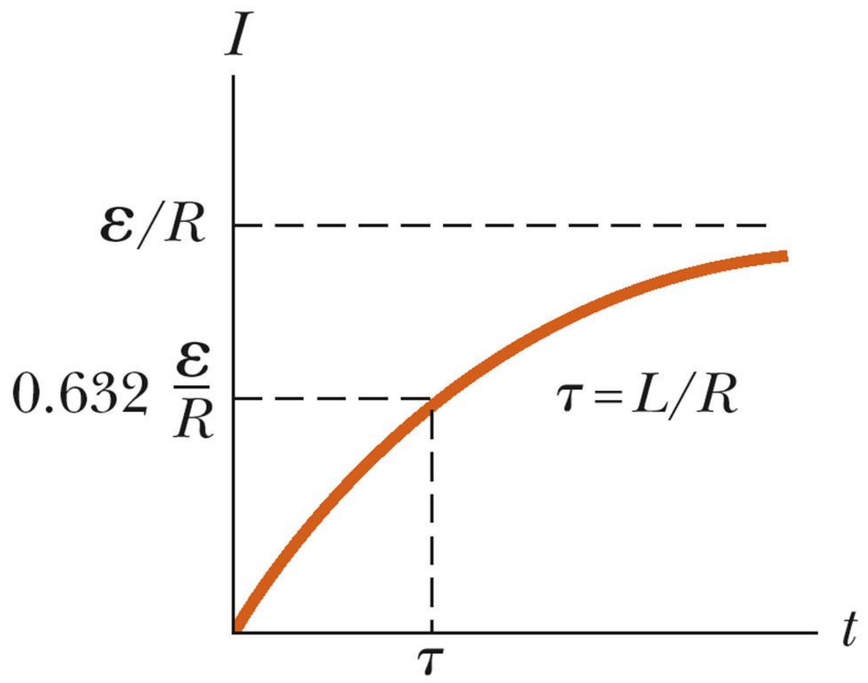
- The expression for the current can also be expressed in terms of the time constant τ, of the circuit:

$$\boxed{\tau = \frac{L}{R}}$$

- The time constant, τ, for an RL circuit is the

  time required for the current in the circuit

  to reach 63.2% of its final value



$$\boxed{I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right)}$$

Energy Stored in a Magnetic Field

$$\varepsilon - L\frac{dI}{dt} - IR = 0 \qquad\qquad I\varepsilon = IL\frac{dI}{dt} + I^2R$$

- **In a circuit with an inductor, the battery must supply more energy than in a circuit without an inductor**

- $I\varepsilon$ **is the rate at which energy is being supplied by the battery**

- **Part of the energy supplied by the battery appears as internal energy in the resistor**

- I^2R **is the rate at which the energy is being delivered to the resistor**

Energy Stored in a Magnetic Field

$$\varepsilon - L\frac{dI}{dt} - IR = 0 \qquad I\varepsilon = IL\frac{dI}{dt} + I^2 R$$

- **The remaining energy is stored in the magnetic field of the inductor**

- **Therefore, *LI* (*dI/dt*) must be the rate at which the energy is being stored in the magnetic field *dU/dt***

$$\frac{dU_L}{dt} = IL\frac{dI}{dt} \qquad \boxed{U_L = \frac{LI^2}{2}}$$

Energy Storage Summary

- **A resistor, inductor and capacitor all store energy through different mechanisms**

- **Charged capacitor stores energy as electric potential energy**

- **Inductor when it carries a current, stores energy as magnetic potential energy**

- **Resistor energy delivered is transformed into internal energy**

Eelectromagnetic induction in Maxwell's equations

- **From Faraday experiment:**

$$\epsilon_{ind} = -\frac{d\psi}{dt} = -\frac{d}{dt}\oint \mathbf{B}\cdot d\mathbf{S}$$

$$\epsilon_{ind} = -\oint \mathbf{E}\cdot\mathbf{dl} = \oint \nabla\times\mathbf{E}\cdot d\mathbf{S}$$

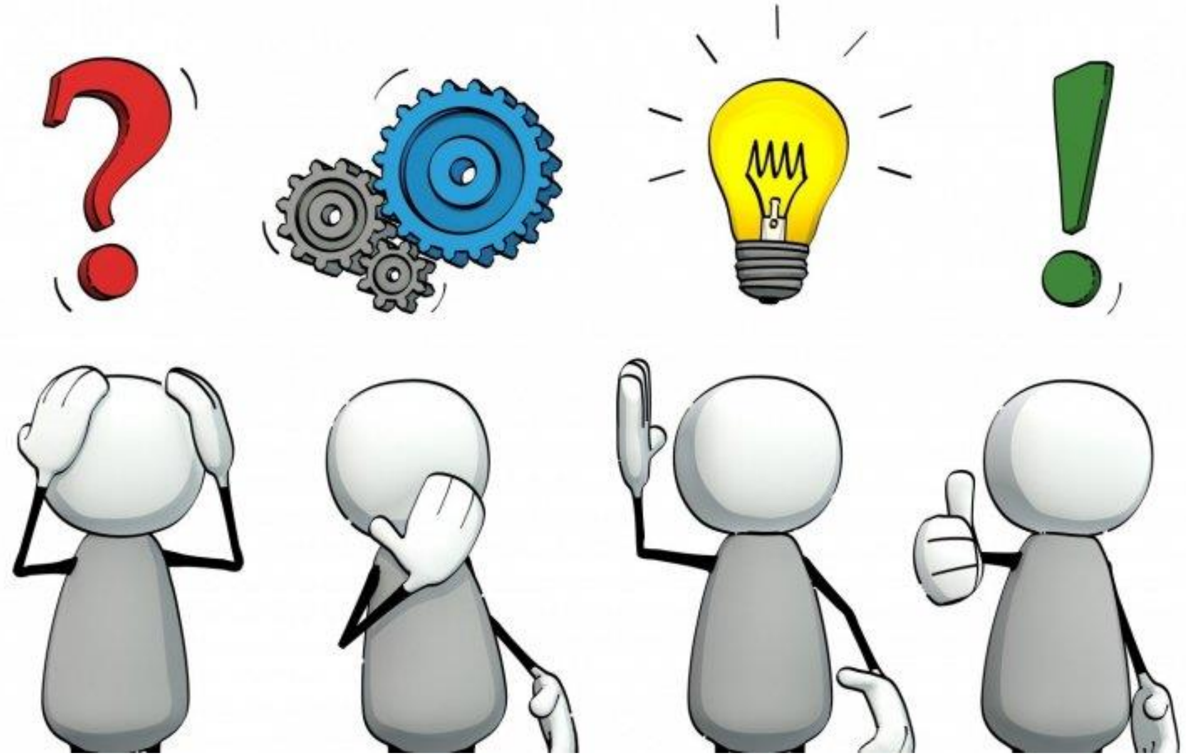
$$\nabla\times\mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

$$\oint \mathbf{E}\cdot\mathbf{dl} = -\epsilon_{ind}$$

- **Unlike the field of static charges:**
  - **The electric field is circular**
  - **Eddy current is a circular current perpendicular to the plane of magnetic field.**

Maxwell's equations

Point Form	*Integral Form*
$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon}$	$Q/\epsilon = \oint \mathbf{E} \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{B} = 0$	$\psi = \oint \mathbf{B} \cdot d\mathbf{S} = 0$
$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$	$\oint \mathbf{E} \cdot \mathbf{dl} = -\epsilon_{ind}$
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$	$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$



Thanks!